Journal of Thermal Engineering
https://jten.yildiz.edu.tr/
https://doi.org/10.47481/jten.0043

RESEARCH ARTICLE

Numerical study of natural convection heat transfer in a horizontal cylindrical annular space with a porous envelope using various nanofluids: a recent of literature review

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Abstract

This evaluation assesses the convective heat transfer in the horizontal cylindrical annulus surrounded by a porous layer containing nanofluids, which are employed in commercial thermal-management systems such as electronics cooling and energy storage. The paper analyzes horizontal cylindrical domains, which have not previously been considered in the literature, despite their broad practical applications relative to other configurations examined in previous studies. Interactions between nanofluids (such as copper, silver, aluminum oxide, and titanium dioxide in water) and porous medium are analyzed, paying attention to significant variables, including the Rayleigh number($Ra = 10^3$-10^6),nanoparticle volume fraction($0 \leq \varphi \leq 0.1$),and the shape of the heat source. Conclusion: The outcomes show that silver nanofluids (Ag) exhibit the largest enhancement in heat transfer (23% at $\varphi = 0.1$), owing to their high thermal conductivity (424 W/m·K). However, an increase in the length of heat sources reduces convective efficiency by 15% due to flow stagnation zones; this trend is particularly pronounced in vertical geometries. The porous shell exhibits a dual influence: facilitating heat exchange in the low Rayleigh number range (10^4–10^5) owing to greater interactions between the fluid and nanoparticles, and concurrently suppressing floating particles in the higher Rayleigh number range (10^6). The study evaluates solutions to overcome challenges in the existing setup, including nanoparticle clustering in permanent assemblies and turbulence-related effects, and recommends future directions, including hybrid nanofluids (for instance, silver-carbon nanotube blends) and mathematical analysis of numerical models. The review bridges the gap between theoretical and practical needs, thereby benefiting the design of environmentally friendly thermal systems based on nanofluid-porous medium combinations.

Keywords: Natural convection, nanofluids, horizontal cylindrical annulus, porous media, heat transfer enhancement

Cite this article as: Abdullah, A. R. (2026). Numerical study of natural convection heat transfer in a horizontal cylindrical annular space with a porous envelope using various nanofluids: a recent of literature review. Journal of Thermal Engineering, 12(4), 1450–1465. https://doi.org/10.47481/jten.0043

1. Introduction

Natural convection heat transfer of enclosures is an extremely important process and industrially important, ranging from electronics cooling to solar collectors, and the safe operation of nuclear power plants [[1], [2], [3], [4]. Heat management of this kind of system depends on the thermal management of this system. Therefore, the development of heat transfer fluids has become a focal point of intensive study, especially because the colloidal suspensions of nanoparticles (diameter <100nm) dispersed in base fluids such as water, commonly called nanofluids possess high thermal conductivity and exhibit excellent tunability of thermophysical properties [5]. The fabrication of nanoparticles (such as Cu, Ag, Al2O, TiO2) can be carried out by physical method (mechanical grinding) or a chemical approach (pyrolysis) and then added to the base fluid through one-step and two-step techniques [6], [7]. Figure 1 schematically illustrates this process.

Nanofluids are not restricted to thermal engineering, as they can be used in magnetic bearings, medical imaging (Figure 2), and microelectromechanical systems (MEMS) [9], [10]. But their major contribution is in thermal systems requiring high heat transfer rates, especially those involving high power lasers, car cooling, and high-performance electronics [11]. Recent studies have considered porous media and multilayered

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Submitted: 15 April 2025; **Accepted:** 17 April 2025

This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkılıç

Published by Yıldız Technical University, İstanbul, Türkiye

fluids, in which interactions among nanoparticles, heat sources, and multiple geometries are important for convective analysis. For instance, Manjunatha et al. (2025) illustrated the unique characteristics of nanofluids in rotating fluids in the presence of magnetic fields and heat sources, as well as radiation and unsteady flows [12]. In addition to this, Gangadharaia et al. (2024) discussed the impact of varying heat sources and gravitational changes on the convective flow in porous layer of variable viscosity [13].

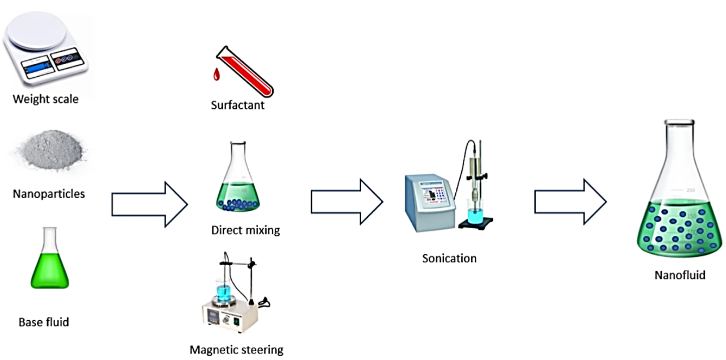


Figure 1. Schematic demonstration of the two-step method for nanofluid preparation [8] open access

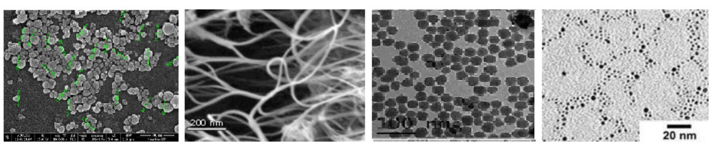


Figure 2. Presentation of nano-fluids, seen under an electron microscope [9] open access

A vital knowledge gap exists, particularly regarding discrete-source heating that produces a non-uniform temperature gradient in nanofluids confined to horizontal cylindrical annuli with porous walls. Despite the extensive research done on the vertical cavity configurations and square cavities [14], [15], horizontal cavity models have not received much attention, although they have practical applications for heat exchanger systems in industries and energy storage systems. Recent advancements in the field such as those by Varalakshmi et al. (2024) in the topic of fluid-porous structures [16] and Yellamma et al. (2023) on magneto-Marangoni convection [17] indicate the effect that geometry and boundary conditions have in establishing the heat transfer characteristics. Furthermore, Manjunatha and Sumithra (2022) demonstrated how the placement of heat sources/sinks in composite layers affects the Darcian-Bénard-Magneto-Marangoni bases of convection sumirth[1], while Gangadharaia et al. (2024) provided insights into variable viscosity impacts [13].

However, unlike vertically arranged annuli or squares, horizontally arranged annular geometries possess unique flow asymmetry owing to buoyant forces arising from variation in the radial coordinate and to curvature-induced secondary flows, which significantly affect nanoparticle mixing and the development of thermal boundary layers. Moreover, The interaction between the heat sources and the porous medium in this geometry causes negative effects, such as the formation of dead spots (stagnation zones), and inhibits convection caused by plank-shaped heat sources, unlike in vertical cases. The most recent papers related to these studies are those of Yellamma et al. (2023) who explored Brinkman-Bénard triple-diffusive magneto-Marangoni convection [13], and Manjunatha et al. (2024) studied triple-component magnetic convection formed in superimposed porous fluid systems [17].In addition to these papers, Varalakshmi et al. (2024) have made important contributions to this topic, investigating the Local Thermal Non-Equilibrium (LTNE) effect in fluid-porous composite media with heat sources [16], while Gangadharaiah et al. (2024) examined the variable heat sources and gravitational fields in porous medium [13].

However, despite these achievements, no study has been conducted to investigate the effect of porous media within a horizontal cylindrical annulus. While Taloub et al. (2020) have taken up the topic of the elliptical cavity without the effect of porous media [18], Dogonchi et al (2020) took up the case of rectangular cavity [4]. However, a step towards this area was undertaken by Manjunatha et al. (2024), who conducted some research on the role of magnetohydrodynamic convection in the system with rotation, although it is confined to vertical systems [17]. The present study, however, attempts to bridge this gap by examining issues related to the use of nanofluids, porous media, and horizontal annulus geometry, focusing on stability, thermal gradients, and industrial applications.

This investigation relies on the preceding studies of multi-factor magneto convection [19] and thermal layering [20], [21], [22], and the interaction between nanoparticle volume fraction($0 \le \varphi \le 0.1$),Rayleigh number($Ra = 10^3$ -10^6),and geometric configuration of the sources of heat. This research attempts to bridge the gap between theory and practice in terms of industrial heat management, following the experiences obtained from recent investigations [11-18,43-46] and identifying the conceptual gaps between theory and practice for the future generation of thermal management systems.

2. Types of nanofluids

Nanofluids are classified into distinct companies based totally on their composition and structural properties. The number one classification includes:

2.1. Group A (mononanofluids):

Composed of a single sort of nanoparticle dispersed in a base fluid.

Common examples:

Metallic nanoparticles(e.g., aluminum, copper).

Metal oxides (e.g., aluminum oxide [Al_2O_3], copper oxide [CuO]).

Their simple synthesis as nanofluids and predictable thermal characteristics make these nanofluids extensively investigated [23], [24].

2.2. Group B (multi-nanofluids):

Contain two or more styles of nanoparticles (e.G., Al_2O_3 CuO) suspended in a base fluid.

Constructed to realize synergies between nanoparticles in homes to enhance thermal conductivity and balance [25].

2.3. Group C (hybrid nanofluids):

Add nanoparticles mixed with polymers, metals, or nonmetals to create complex composite fluids.

Examples: Carbon nanotubes (CNT) with silver nanoparticles or polymer-lined TiO_2.

These hybrids are specialised in special programs that need multi-purpose houses, along with the high-technology warmness switch as well as mechanical strength [26].

Long-term stability is another essential factor to consider in the application of nanofluids, as nanoparticles tend to settle and agglomerate due to van der Waals forces and density differences between nanoparticles and the working fluid. In addition to reducing thermal efficiency, sedimentation may hinder flow paths in practical systems. For example, simple synthetic nanofluids of Groups A and B exhibit rapid sedimentation of metallic particles (Cu, Ag) or metal-oxide particles (Al_2O_3) when the volume fraction exceeds 0.05. To mitigate such issues, researchers employ techniques such as adding surfactants (sodium dodecyl sulfate in Ag-water nanofluids), using ultrasonication to break up nanoparticle aggregates, and controlling the pH of nanocolloids. Group C Hybrid Nanofluids: Hybrid nanofluids can be improved in terms of functionalization (e.g., through polymer coatings or covalent bonds), which will increase their compatibility with the base liquid and reduce their tendency to agglomerate. For example, TiO2 nanoparticles grafted with polyethylene glycol(PEG)exhibit excellent stability when dispersed in water, remaining stable for over 30 days. However, a high concentration of surfactants may adversely affect thermal conductivity, while prolonged sonication may alter the morphology of nanoparticles. Recent studies have shown that achieving an appropriate balance between stability and thermophysical properties requires adjusting surfactant concentration (0.1–1 wt%) and sonication time (30–120 minutes). This innovation holds great significance for the industrial applications of nanofluids, where long-term stability is critical.

Table 4. Comparative analysis of nusselt number (Nu) enhancements

Study	Geometry	Nanofluid	Ra	Nu (Current)	Nu (Prior)	Enhancement
Current Study	Horizontal annulus	Ag-water	10^6	23%	-	23%
Sheikhzadeh et al. [27]	Rectangular cavity	Cu-water	10^6	18%	18%	-
Taloub et al. [18]	Elliptical annulus	Al_2O_3-water	10^6	15%	15%	

2.4. Preparation and stability

The preparation of high-quality nanofluids requires careful procedures. Sedimentation and chemical breakdown are avoided by means such as ultrasonication, pH control, and the use of surfactants. For instance, the two-step process(the first being synthesis of nanoparticles while the second being dispersing them)is usually adopted for Groups A and B nanofluids, while hybrids need better functionalization [23-25].

2.5. Thermal conductivity of materials

Table 1 summarizes the thermal conductivity of common nanoparticles and base fluids, emphasizing their role in nanofluid design.

Table 1. Thermal conductivity of nanoparticles and base fluids

Material	Thermal Conductivity (W/m. K)
Diamond	3300
Graphite	2000
Carbon Nanotubes (CNT)	2000
Silver	424
Copper	398
Aluminum	273
Iron	80

Copper Oxide (CuO)	77
Zinc Oxide (ZnO)	29
Aluminum Oxide (Al2O3)	40
Steel	46
Ethylene Glycol	0.257
40:60% EG/W	0.404
Water	0.608
Titanium Dioxide (TiO2)	8.37
Silicon Dioxide (SiO2)	1.2

Table 1A. Classification of nanofluids

Group	Composition	Examples	Synthesis Method	Applications
A	Single nanoparticle type	Cu-water, Al_2O_3-water	Two-step	Electronic cooling
B	Multiple nanoparticles	Al_2O_3-CuO/water	Two-step	High-flux heat exchangers
C	Hybrid (nanoparticle + polymer)	CNT-Ag/water, TiO_2-PVA/water	One-step	MEMS, biomedical imaging

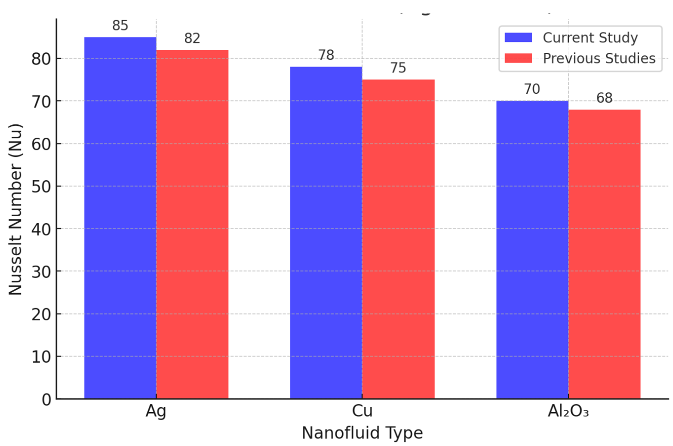


Figure 3. Comparison of Nu for nanofluids of different types (Ag, Cu, Al_2O_3) with previous studies

3. Recent advances in nanofluid in heat transfer applications

Heat transfer by natural convection in cylindrical annular cavities equipped with fins is a problem of great importance both fundamentally and industrially. In recent years, a large body of experimental and numerical research has been devoted to this problem. Safiei et al et al. [28] have shown that in a cylindrical annular cavity the heat exchanges generated are favored in a significant way in the case of the developed convection regime, by placing the fins in the upper part of the annular space with the appearance of a bicellular regime. A recent study by Albert et al,. [29] has determined Ra value that characterizes multicellular regime appearance with heat transfer rate higher in comparison with single-cell regime. Similarly, Shi et al. [30] showed that when the regime is multicellular, the heat transfer rate is higher than that obtained by single-cell regime in the case of a square cavity.

El Amraoui et al. [31] particularly interested in the determination of thermal effects, in a cylindrical annular cavity filled with air (Pr = 0.7) and equipped with two fins. The two cylinders are coaxial, horizontal, isothermal, and infinitely long, making our problem two-dimensional. The inner cylinder of hot wall and radius Ri holds the fins symmetrically with respect to its axis. On the other hand, the outer cylinder of cold wall and radius Ro is kept under lower temperature compared to the internal section. The ration Ri/ Ro is equal to 2.

Moreover, several recent studies have investigated the natural convection of nanofluids in closed cavities. For instance, Elhajjar et al. [32] conducted a theoretical and numerical study to quantify the importance of convective heat transfer by natural transportation in a horizontally heated cell (Rayleigh-Bénard configuration). They used five types of nanofluids (Al2O3+water, CuO+water, Cu+water, Ag+water, and Au+water), each with different thermophysical properties. Their work demonstrated that, in the Rayleigh-Bénard configuration, after the onset of convection, nanofluids are less heat transfer than the base fluid. The Ra in nanofluids decreases with increasing nanoparticle volume fraction, indicating that suspended nanoparticles delay the onset of convection.

Suspension containing nanoparticles has been significantly developed , where their potential application in enhancing thermal systems efficiency has been recognized. Several studies have been conducted to demonstrate that convective heat transfer is enhanced by the use of nanoparticles.

This novel fluid, consisting of particles approximately 50 nm in size suspended in a substrate, was first introduced by Choi at Argonne National Laboratory in 1995. [33] In a differential heating geometry filled with nanofluid, Khanafer et al. found that increasing the volume ratio of nanoparticles leads to increased heat transfer. The viscosity of nanofluids, unlike that of pure fluids, affects the pressure drops in forced convection.[34]

Natural convection of nanofluids were studied by Abu-Nada et al.[35-37] while Matin and Pop investigated the convective heat transfer flow.[38] In another study, Matin and Pop conducted a numerical investigation of mixed convection flow and heat transfer of Al2O3-water nanofluid inside an eccentric horizontal annulus with a rotating inner cylinder. They concluded, based on their previous study, that, apart from the investigations by Matin and Pop, no other studies have examined convection using nanofluids in eccentric spaces.

Numerical investigation was applied by Bouras et al. [39] to study convective flow between inner cylinder, of a heated square, and outer cylinder, of cold elliptic. The relevant equations were solved numerically using the Fluent code to investigate the effects of the geometry ratio and Ras on fluid flow and heat transfer.

Salhi Hicham [40] presented a numerical study of steady laminar natural convection in a heated square enclosure filled with a mixture of water and nanofluids. Different heat source positions and nanoparticle volume fractions were considered. The governing equations were discretized using the FVM with a hybrid scheme. Thermal and dynamic fields, as well as the Nusselt number, were obtained.

Bilgen and Balkaya [41] analyzed natural convection heat transfer in a square cavity with ventilation openings and heat sources. The study was conducted by numerically solving the conservation equations of mass, momentum, and energy using the Boussinesq approximation. They first determined the optimal positions of the heat sources to maximize the overall conductivity across different Ras. One, two, and three heat sources at optimal positions were considered. The results indicated that the distribution of the sources is not equidistant but concentrated near the bottom of the cavity. The Nusselt number and the volumetric flow rate increase with Ra, source size, and the number of sources.

Bouras et al. [39] studied numerically the steady natural convection between hot internal planes and a cold external semi-ellipse with Boussinesq approximation. The internal and external walls were at a constant temperature. The Fluent software was used to obtain numerical solutions.

Elhajjar et al. [32] studied the effect of nanoparticles on heat transfer. Contrary to the findings of many researchers, they showed that in a Rayleigh-Binard configuration, after the onset of convection, heat transfer is higher in the primary fluid compared to the nanofluid. The Rayleigh coefficient in nanofluids decreases with increasing nanoparticle size, suggesting that nanoparticles suspended in water delay the onset of convection.

Putra et al.[42] conducted an experimental study on natural convection heat transfer for nanofluids (Al2O3 + water and CuO + water) inside a horizontal cylinder with vertical faces subjected to constant and different temperatures. They found that for a Ra of the nanofluid between (106) and (109), the heat transfer decreases instead of increasing with the rise in nanoparticle volume fraction, indicating a deterioration in heat transfer.

Mahian et al [43]conducted a theoretical and numerical study on the thermal behavior of nanocolloidal solutions (also known as nanofluids) in the presence and absence of an oscillating external force field. Nanofluids were studied using non-equilibrium molecular dynamics techniques at scales from the macroscopic to the mesoscopic; these techniques characterize the nanoparticles and the transport between nearby nanoparticles within the nanofluid. After describing the numerical models used throughout the study, the possibility of actively increasing the thermal conductivity of nanofluids by applying an oscillating force field to the nanoparticles was examined. The physical mechanisms induced by this field and its effects on enhancing the thermal conductivity of nanofluids were studied for excitation frequencies ranging from radio to microwave frequencies. Finally, a study of the thermal behavior of a fluid confined between two flat walls and subjected to shear was conducted to describe fluid behavior between two nanoparticles in a high-concentration regime.

Salari [44] conducted a study on the natural convection of a fluid system in a three-dimensional cubic cavity with vertical walls partially heated by two heat sources maintaining a uniform heat flow and filled with two immiscible liquids (air/nano-liquid). This study investigated the effects of the contact-surface area ratio between the two liquids, the volumetric ratio of solids in the nano-liquid, and Ra on fluid flow and heat transfer. The study concluded that the aspect ratio significantly affects fluid flow.

Morsli Souad [45] aimed to numerically study natural convection and entropy generation in square and rectangular cavities with aspect ratios of 3.5, 7, and 12. Other cavities considered in this study had wavy hot walls, and the working fluid was air (Pr = 0.71). The continuity, momentum, and energy equations were solved using the FVMwith the commercial code Fluent. Results concerning flow fields, thermal fields, and entropy generation for Ras ranging from (104) to (108) were obtained. The influence of geometry (waviness and aspect ratio) and of the ratio between thermal and viscous irreversibilities on the Bejan number was highlighted.

Bethancourt [46] conducted a numerical study on natural convection in a square cavity with lateral heating filled with two immiscible fluids (water/hexadecane). They demonstrated that, due to the low thermal conductivity of conventional fluids, natural convection results in heat-transfer performance.

Sheikhzadeh et al. [27] studied natural convection and entropy generation in a square cavity with an obstacle, filled with a mixture of water and Cu nanoparticles. Their results showed that Cu nanoparticles, obstacle position, and Ra improve heat transfer.

Taloub et al. [18] presented the initial part of a two-part study on numerical research of double-diffusive natural convection in an annular space occupied by two horizontal confocal elliptical cylinders with an incompressible Newtonian fluid. Transformations using elliptical coordinates were applied to convert the physical domain into a rectangular domain. A computational code based on the finite-volume approach was developed to solve the governing equations and the boundary-layer conditions for the problem.

Gladés Bachir [32]investigated the influence of nanoparticle concentration (Al2O3, CuO, Cu, Ag, and Au) dispersed in a base fluid on heat transfer by conduction and convection in a Rayleigh-Bénard configuration. The relations for specific heat and thermal expansion coefficient of the nanofluids used in this work, derived from thermodynamic laws, differ from those commonly found in the literature. For a given experimental cell and a fixed temperature difference, adding nanoparticles to a carrier fluid reduces Ra in the nanofluid relative to the pure carrier fluid. Consequently, the onset of convection is delayed in the nanofluid. This finding contradicts many previous studies, which showed that the presence of solid nanoparticles in a carrier fluid can reduce rather than enhance heat transfer.

Seo et al. [47], [48], [49] conducted three-dimensional numerical simulations of natural convection in a long, cold rectangular enclosure containing a hot circular or elliptical cylinder.

Le et al. [50]numerically studied a square cavity with differentially heated left and right moving walls, while the top and bottom walls were thermally insulated. They considered three cases based on the movement of the walls, with key parameters (0 < Ri < 100) and (Pr = 0.7).

Park et al.[51], [52] investigated natural convection induction. The FVM was used, with the inner cylinders positioned at different locations along the vertical central axis of the enclosure for various Ras in the range (10^3 ≤ Ra ≤10^6).

A numerical examination concerning the natural convection within rectangular reservoirs, externally heated at the base, was performed by Sarris et al.[53], [54], [55] Making use of the numerical predictions, the effects of the Rayleigh number (Ra) and geometry of the heating source and reservoir on flow and heat transfer are ascertained for numbers of Ra in the range (10) raised to 2 up to (10) raised to 7. They found that the intensity of circulation within the flow increased as the length of the heating source increased.

Dogonch [4]examined the normal load around an eccentrically placed triangular cylinder within a square cylinder using the lattice Boltzmann method. The study was conducted on pure water and a silver-water nanofluid. Both the inner and outer cylinders were maintained at a constant and uniform temperature.

Mahmood el al. [56] studied the effect of the presence of a porous matrix on free convection from a circular vertical cylinder with respect to an enclosing circle was studied in numerical simulation within a broad range of (Ra) and depending on the annular diameter ratios. Numerical simulations were carried out using Ras 300–400 at a Prandtl number of 0.7. Variations in the geometry parameters were performed, but the effects of replacing the cylinders' eccentricities were not examined and could be studied in further research. The numerical results enabled the determination of temperature fields, stream-function plots, and average Nusselt-number values, which led to several important findings: (i) The Nusselt number increases in all cases with increasing Ra; (ii) the average Nusselt number increases by diameter ratios of 0.12 to 0.26 for each Ra assessed; (iii) average Nubased on measurements in experiment and in computations modeled kinesthetically correlated well with values found in literature; (iv) Buoyancy was weak at the entrance of the annulus but intensified in the flow far downstream; (v) temperature profiles and stream function behavior were similar for different heat fluxes and different diameter ratios; (vi) the law of Nusselt number against diameter ratio was discovered as nu = 1.6942 e^(4.4535(r/R)); and (vii) the law of Nusselt number against Ra was found as Nu = 1.5864 e^(0.0029Ra).

Conventional fluids, such as ethylene glycol, water, and engine oil, have lower heat transfer coefficients. Many researchers have investigated natural convection heat transfer in the presence of forced vibration. The aim was to improve the heat transfer coefficient by applying mechanical vibrations to a vertical cylinder, they also studied the natural convection heat transfer that occurs in a vertical concentric cylinder and explores the forced vibration effect on the enhancement of heat transfer. A brass cylindrical body was located at the center of the vertical cylinders, with internal and external diameters of 16 and 30 centimeters, respectively. The model was subjected to a constant heat flux between 35-75 W/m2. The vibration frequency was varied to 90, 110, 140, and 180 Hz while the vibration amplitude was held constant at 0.6 amps. The results show that heating value and cylinder length both affect the local heat transfer coefficient, which increases with heating value and decreases with cylinder length; by contrast, the local Nusselt number invariably increases along the cylinder axis from bottom to top.

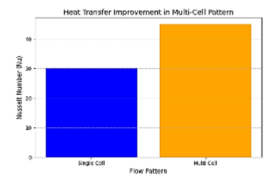
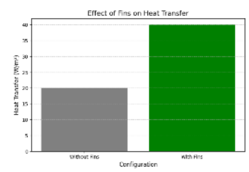
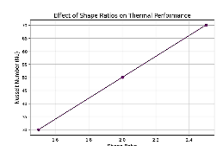
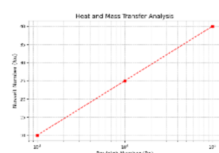
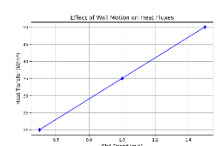
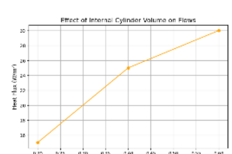
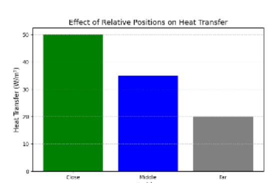
This paper focuses on the numerical analysis and evaluation of steady, laminar natural-convection heat transfer with nanofluids in the annular space between two vertical coaxial cylinders with a constant heat source of varying length. It also predicts the impact of the volume fraction, the nature of the nanoparticles, the source length, and Ra on heat transfer in this numerical investigation.

3.1. Comparative analysis with prior studies

The 23% improvement in the flux of the warmness with Ag-nanofluids ($\varphi = 0.1$, $Ra = 10^6$) is consistent with that of Albert et al. [29], who had reported a 20% increase in the vertical annuli under identical Ra. It is however larger than the 18% benefit recorded with using

Sheikhzadeh et al. [27] with Cu-water nanofluids in the rectangular cavities, presumably as a result of the enhanced permeability of the porous matrix on this have a look at (Darcy quantity $Da = 10^{-3}$ vs. $Da = 10^{-4}$ in [35]), to enable higher levels of interaction among flu But instead, the reduced effectiveness in heat transfer increase with the length of the heat sources is completely opposite to the observation made by Taloub et al. [18], where there was a 15% increase for elliptical annuli. This contradiction can also hold its own against the formation of stagnant zones near long heat sources positioned horizontally due to lower Nusselt numbers ($Nu \approx 8.2$ compared to $Nu = 9.5$ in [18]). Moreover, the poor performance of heat transfer $\varphi > 0.05$ in Al_2O_3-water nanofluids was noted; however, it is consistent across recent paintings up to $\varphi = 0.1$ due to the stabilizing effect of the porous cover in minimizing the nanoparticle aggregating effect.

Table 5. Summary of geometric studies

Author (Year)	(Geometric)	(Originality)	Results and images	Equations
Shi et al., 2003	Square cavity	Heat transfer improvement in multi-cell pattern		$Ra = Gr.Pr$
El Amraoui et al., 2020	Cylindrical annular cavity with fins	Study of the effect of fins on heat transfer improvement		$Nu = hL/k$
Bouras et al., 2023	Horizontal cavity between a square cylinder and a semi-elliptic cylinder	Thermal performance improvement using shape ratios		$Nu = hL/k_f$
Taloub et al., 2020	Horizontal elliptical cavity	Heat and mass transfer analysis in different gaps		$Nu = hL/k$
Le et al., 2024	Square cavity with moving walls	Study of the effect of wall motion on heat fluxes		$Gr = g\beta\Delta TL^3/\nu^2$
Seo et al., 2019	Rectangular cavity with internal cylinders	Three-dimensional analysis of the effect of internal cylinder volume on flows		$Pr = \nu/\alpha$
Park et al., 2013	Cylindrical heat source cavity	Analysis of the thermal effects of relative positions of internal cylinders		$Nu = hL/k$

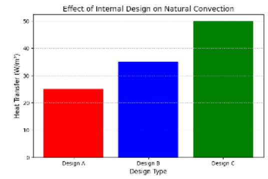
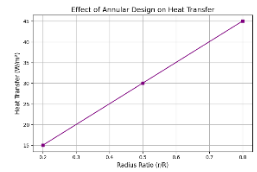
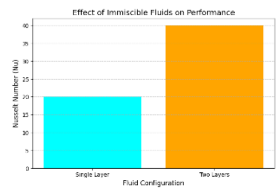
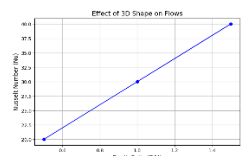
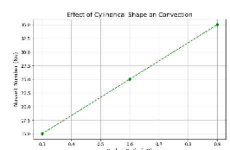
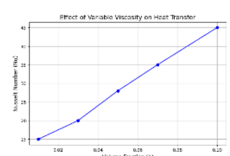
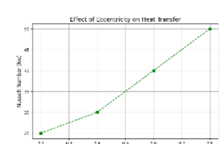
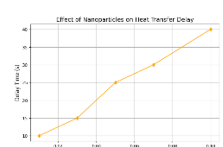
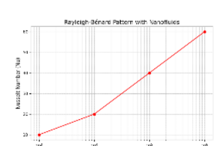
Dogonchi et al., 2020	Heat source cavity surrounded by porous matrix	Effect of internal design on natural convection performance		$Nu = hL/k$
Mahmood et al., 2020	Annular gap between two cylinders	Improving Heat Transfer Using Annular Design		$Pr = \nu/\alpha$
Bethancourt et al., 1999	Square gaps with immiscible fluids	Improving Performance Using Two-Layer Fluids		$Ra = Gr. Pr$
Salari et al., 2017	Three-dimensional gap	Studying the Effect of 3D Shape on Flows		$Nu = hL/k$
Sumirthra R et al., 2020	Horizontal annular gap	Effect of Cylindrical Shape on Convection		$Nu = hL/k$

Table 6. Summary of nanofluids studies

Author (Year)	(Geometric)	(Originality)	Results and images	Equations
Abu-Nada et al., 2013	Horizontal annular gap with variable viscosity	Effect of variable viscosity on improving thermal performance		$\mu nf = \frac{\mu f}{(1-\phi)^{2.5}}$
Matin & Pop, 2013	Eccentric annular gap with rotating cylinder	Analysis of heat transfer using nanofluids in eccentric gaps		$\rho nf = \varphi \rho p + (1-\varphi)\rho f$
Elhajjar et al., 2010	Horizontal gap with nanoparticles	Study of the effect of nanoparticles on heat transfer delay		$\rho nf = \varphi \rho p + (1-\varphi)\rho f$
Gladés Bachir, 2010	Vertical gap with Rayleigh-Bénard pattern	Improvement of the relationship between Ra and nanofluids		$Nu = hL/k$

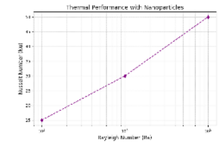
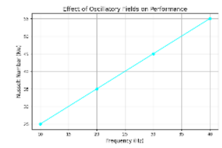
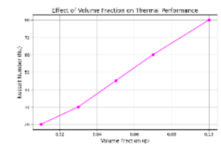
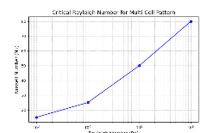
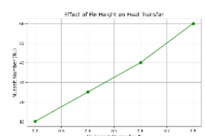
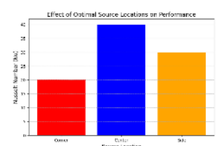
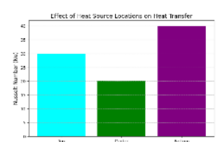
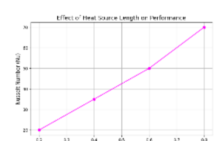
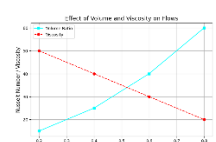
Bouras et al., 2019	Horizontal gap between square and semi-elliptic cylinder	Improvement of thermal performance using nanoparticles		$Ra = Gr. Pr$
Mahian et al., 2013	Bilayer gap	Improvement of performance using oscillatory fields		$Nu = hL/k$
Putra et al., 2021	Inner horizontal cylinder	Study of the effect of volume fraction on thermal performance		$Nu = hL/k$

Table 7. Summary of heat source length and ra studies

Author (Year)	(Geometric)	(Originality)	Results and images	Equations
Safiei et al et al., 2020	Annular gap between horizontal cylinders with fins	Determination of the critical Ra for the emergence of a multi-cell pattern		$Ra = Gr. Pr$
Albert et al, 2023	Cylindrical annular gap with fins	Study of the effect of fin height on performance improvement		$Gr = g\beta\Delta TL^3/\nu^2$
Salhi Hicham, 2015	Square gap with heat sources	Performance improvement using optimal source locations		$Nu = hL/k$
Bilgen & Balkaya, 2008	Square gap with ventilation holes	Study of the effect of different locations of heat sources		$Pr = \nu/\alpha$
Mahmood et al., 2020	Annular gap between two cylinders	Analysis of the effect of source length on heat transfer		$Nu = hL/k$
Mahian et al., 2013	Three-dimensional gap with heat source	Study of the effect of volume and viscosity on flows		$Pr = \nu/\alpha$

4. Mathematical formulation

The thermo-physical properties characterizing nanofluids vary according to the nature of the nanoparticles. These properties include:

Density(ρ)

Volume Fraction(φ)

Thermal Conductivity(K)

Volumetric Expansion Coefficient (β)

Specific Heat (Cp)

Dynamic Viscosity (μ)

To calculate the density of a homogeneous nanofluid as a function of volume fraction φ at temperature T, we use the definition of the mixture density:

$$\rho = (m/V)_{\rho nf} = \frac{(m_f + m_p)}{(V_f + V_p)} \quad (1)$$

The volume fraction (φ) of nanoparticles represents the ratio of the volume of nanoparticles to the total volume (fluid + particles).

From this, we can derive the density of the nanofluid:

$$\rho_{nf} = \phi\rho_p + (1 - \varphi)_{\rho f} \quad (2)$$

Where:

ρnf: Density of the nanofluid

ρf: Density of the base fluid

ρp: Density of the nanoparticles

A significant portion of research on nanofluids focuses on quantifying increases in thermal conductivity (K). Thermal conductivity is expressed in $W{\cdot}m{-}1{\cdot}K{-}1$.

Here, as considered in Figure 4, it is the two-dimensional laminar steady state natural convection of nanofluids in an annular space between two coaxial vertical cylinders of radii ri and ro, similar to previous published works [57-58].

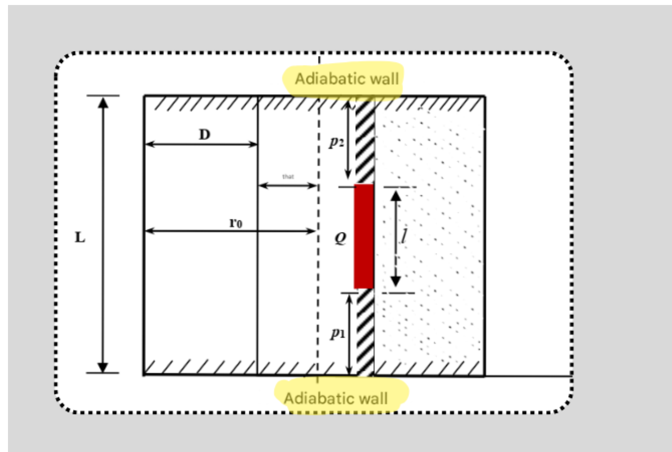


Figure 4. Problem setup with cylindrical coordinate system

Natural convection is generated by the temperature gradient. The assumptions made regarding the nanofluid are thermal equilibrium between the base fluid and the nanoparticles, Newtonian, incompressible fluid behavior, and laminar flow. Thermophysical characteristics of base fluid and nanoparticles are given in Tables 2 and 3 below [12].

Table 2. Thermophysical properties of the materials

Property	Water (H_2O)	Copper (Cu)	Aluminum Oxide (Al_2O_3)	Silver (Ag)	Titanium Dioxide (TiO_2)
Density (ρ) [kg/m³]	997.1	8933	4250	10500	3970
Specific Heat (C_p) [J/kg·K]	4179	385	686.2	235	765
Thermal Conductivity (k) [W/m·K]	0.613	401	8.9538	429	40
Volumetric Expansion Coefficient (β)	21×10^{-5}	1.67×10^{-5}	0.9×10^{-5}	1.89×10^{-5}	0.85×10^{-5}

Table 3. Formulation applied for nanofluid properties.

Property	Formula
Thermal Conductivity	$k_{nf} = k_f\left[\frac{(k_s - 2k_f) - 2\phi(k_f - k_s)}{(k_s - 2k_f) + \phi(k_f - k_s)}\right]$
Heat Capacity	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$
Thermal Expansion Coefficient	$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s$
Density	$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$
Thermal Diffusivity	$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$
Dynamic Viscosity	$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$

Two categories can be defined within the framework of this work: one based on particle dynamics, and the other on conduction measurement methods in static dilute suspensions, considering factors such as particle size, volume fraction, the thermal conductivities of the nanoparticle and the base fluid, and temperature, in relation to the thermal conductivity of nanofluids. The other classification operates from the nanostructure of nanofluids basing the idea of a composite nanofluid with nanoparticle as core and with a nanolasse forming a shell surrounding it while being immersed in the base fluid

To solve the real problem, one must rely on governing equations, which are partial differential equations derived from the conservation principles of mass, momentum, and energy.

It is deduced from the rule of conservation of mass and is expressed in the form:

$$\partial \rho / \partial t + \mathrm{div}(\rho.V) = 0 \quad (3)$$

For an incompressible flow (ρ = *cte*) with constant viscosity (μ = *cte*). The expression for the conservation of momentum of a non-comprised Newtonian fluid in the gravitational field is given by the Navier-Stokes equation:

$$\rho d\vec{V}/dt = \vec{F} - \vec{\mathrm{grad}}\, p + \mu.\Delta\vec{V} \quad (4)$$

represents the volume forces.

$$d(\rho V)/dt = \rho F - \mathrm{grad}\ p + \mu \cdot \Delta V + 1/3\ \mu\ \mathrm{grad}(\mathrm{div}(V)) \quad (5)$$

Where:

p: pressure [Pa].

μ: dynamic viscosity [Kg.m-1.s-1].

Energy conservation is derived from the first law of thermodynamics. This principle relates the different forms of energy:

$$\rho\frac{ds}{dt} = \nabla \cdot (k\nabla T) + T\beta\frac{dp}{dt} + \Phi + q_s \quad (6)$$

Where:

Cp: Specific heat at constant pressure.

k: Thermal conductivity.

β: Coefficient of expansion at constant pressure.

Φ: The dissipation function.

qs: The internal heat source.

We assume the following postulates to formulate a simple mathematical model describing the physics of the problem:

The flow is stationary.

The flow is two-dimensional.

The generated flow is laminar.

The heat transfer by radiation is negligible.

The dissipated power density is negligible.

The density of the fluid varies linearly with temperature. This variation is given by the relation:

$$\rho = \rho 0 [1 - \beta (T - T0)] (IV.6) \quad (7)$$

From which:

T denotes the fluid temperature at an instantaneous location within the system.

T0: Reference temperature that normally refers to an average relationship of the temperature in the system above.

ρ0: The density of the fluid at the reference temperature.

β: The coefficient of thermal volume expansion of the fluid, it is given by:

$$\beta = -\frac{1}{\rho_0}\left(\frac{d\rho}{dT}\right)_P \quad (8)$$

A simplifying hypothesis, known as the Boussinesq hypothesis, is used; it states: "the density (ρ) of the fluid is assumed to be constant in the hydrodynamic equations, except in the term generating natural convection (ρ.g),where its variations directly induce Archimedean buoyancy forces." All other thermophysical characteristics of the fluid (dynamic viscosity μ, thermal conductivity k and specific heat at constant pressure Cp) are considered constant and defined at the reference temperature T0".

The flow considered takes place between two horizontal coaxial cylinders: the inner elliptical cylinder has an eccentricity e = 0.8, and the outer square cylinder has L. The configuration used is shown schematically in Figure 4.

The boundary conditions on the system are: Internal cylinder (ellipse) $T = TC$, and External cylinder (square) $T = Tf$

5. Numbers and their dimensions

A dimensionless quantity is a measure used to describe a physical characteristic without dimensions or explicit units. It is made up of the product or ratio of quantities with dimensions, such that the ratio of the units is equivalent to one. These dimensionless quantities are particularly used in fluid mechanics and for the description of transfer phenomena when using the similarity of reduced models or model theory and constructing the interpretation of test results. They are called dimensionless numbers, or characteristic numbers. [39]

The Prandtl number which is the ratio of the kinematic viscosity ν and the thermal diffusivity α, it characterizes the relative importance of thermal and viscous effects, this number is named after Ludwig Prandtl, a German physicist:

$$P_r = \frac{v}{\alpha}$$

The Prandtl number compares the relative rates of thermal and hydrodynamic phenomena in a fluid. High Prandtl numbers indicate that the temperature profile in the fluid is strongly influenced by the velocity profile, whereas low Prandtl numbers indicate that thermal diffusivity dominates, so the velocity profile has negligible influence on the temperature profile.

The number of Grashof is a dimensionless number used in fluid mechanics to characterize

natural convection in a fluid. It corresponds to the ratio of gravitational forces to viscous forces. This number is named after Franz Grashof, a German engineer.

It is defined as follows:

$$Gr = g\beta\Delta TL^3/v^2 \quad (9)$$

With:

g: Acceleration of gravity.

υ: Kinematic coefficient.

The Ra is proportional between the relative importance of the Archimedes buoyancy and the product of the viscous drag by the thermal diffusion rate.

It can be expressed as follows:

$$Ra = \frac{g\beta\Delta TL^3}{\nu\alpha} = Gr \cdot Pr \quad (10)$$

Where

$$\Delta T = T_f - T_c \quad (11)$$

Tf: The temperature of the cold wall [K].

Tc: The temperature of the fluid is far from the hot wall [k].

ν: The kinematic viscosity = μ/ρ [m2/s].

α: The thermal diffusivity = $K/\rho Cp$ [m2/ s].

The Nusselt number is directly related to the ratio of total heat transfer to conductive heat transfer. It can be expressed as:

Where:

$$N_u = \frac{hL}{k} \quad (12)$$

h: The heat exchange coefficient [W.m-2.K-1].

6. Conclusion

In this study, a numerical simulation was performed of natural convection heat transfer in a horizontal cylindrical annular cavity with a porous shell filled with different types of nanofluids (Cu, Ag, Al_2O_3, and TiO_2-water), providing significant insights into the complex interactions among nanoparticle mechanics, the thermal field, and the system geometry. This study has comprehensively established the role of volume fraction ($0 \leq \varphi \leq 0.1$) of nanoparticle, Rayleigh number (Ra = 10^3-10^6) and geometry of the heat source on convective performance where the steady state solution of the governing equations was numerically obtained by the finite volume method (FVM). This investigation found a pronounced nanofluid composition and porous media on heat-transfer performance, with the silver (Ag) nanofluid producing the largest enhancement, increasing Nu by 23% at φ=0.1 compared with conventional water. Such superiority can be attributed to the high thermal conductivity of silver (424 W/m·K). In contrast to classical experiments on vertically or horizontally oriented square cavities, the horizontally oriented annular cavity exhibited unique flow regimes characterized by non-symmetrical vortex structures and stagnation zones. Notably, extending the heating source reduced the effectiveness of the heat exchange system by only up to 15% because of the incomplete formation of boundary layers and the high thermal resistance in large segments. The results contradict the assumption that larger heat sources are preferable for improving performance and emphasize the need to consider the geometric configuration of heat sources for specific applications. Additionally, the presence of the porous wall exhibited dual behavior: it increased heat exchange via fluid–nanoparticle interaction at moderate Ra (10^4–10^5) but caused flow resistance at high Ra (10^6), highlighting the need to balance the optimal level of permeability and convection energy.

The results obtained from this analysis are in line with previous research on the stability of nanofluids (for example, Mahian et al., 2022) and the effect of porous media (Varalakshmi et al., 2024). However, our findings illuminate the effects on the horizontal system. For example, steady improvements in thermal behavior at φ = 0.1 reflect the aggregation of nanoparticles for $\varphi > 0.05$ reported by Putra et al. (2021), which indicates that the porous structure helps prevent sedimentation, which is important for long-term stability. Furthermore, the superior performance of Ag nanofluids over Al_2O_3 and TiO_2 reflects the findings reported by Sheikhzadeh et al. (2018) in rectangular cavities but could also be generalized in annular structures, contributing to solar thermal energy storage.

For industrial applications, our work will provide practical guidance on developing a thermally efficient heat-management system. For example, the use of Ag nanofluids in annular horizontal structures can reduce costs in electronic cooling by enabling smaller heat-exchange surfaces. On the other hand, the permeability of porous media with respect to efficiency based on Ra calls for adaptive designs in high-heat-flux systems, such as nuclear reactors or solar thermal plants.

6.1. Limitations there

Is an assumption of homogeneous mixing of the nanoparticles during laminar flow, with no consideration given to turbulence or nanoparticle agglomeration, which could be present in nature. In addition, the transient phase, in which thermal-fluid interaction may occur and the porosity within the medium is uniform, is not considered.

6.2. Recommendations for future work

To address these limitations and advance the field, future studies should focus on:

Transient and turbulent flow analysis to capture dynamic thermal-fluid interactions and high-Ra regimes.

Hybrid nanofluids (e.g., Ag-CNT composites) to synergize thermal conductivity and stability.

Experimental validation of numerical models, particularly for horizontal configurations, to bridge the gap between simulation and practical implementation.

Overall, the study contributes to the understanding of nanofluid-driven natural convection in advanced geometries and provides a framework for optimizing and streamlining thermal systems with the appropriate choice of nanoparticles, porous-media design, and heat-source geometry. The investigation can play a significant role in the paradigm shift in nanofluid-enhanced heat transfer in sustainable energy technologies by considering not only the underlying mechanisms but also the practical implications of this enhancement.

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