YTUP

Journal of Thermal Engineering

https://jten.yildiz.edu.tr/

https://doi.org/10.47481/jten.0024

RESEARCH ARTICLE

Wake effect in wind farms: comparative study of models and their impact on power production

Cherif Saib[1], Said Zergane[2], Salah Amroune[3], Ahmed Belaadi[4], Amar Berkache[5], Barhm Mohamad[6,*]

[1]Mechanical Engineering Department, University of Mohamed Boudiaf, M'sila, 28000, Algeria
[2]Mechanical Engineering Department, University of Mohamed Boudiaf, M'sila, 28000, Algeria
[3]Mechanical Engineering Department, University of Mohamed Boudiaf, M'sila, 28000, Algeria
[4]Department of Mechanical Engineering, Faculty of Technology, University 20 August 1955, Skikda, El-Hadaiek, Skikda, 21000, Algeria
[5]Mechanical Engineering Department, University of Mohamed Boudiaf, M'sila, 28000, Algeria
[6]Department of Petroleum Technology, Koya Technical Institute, Erbil Polytechnic University, Erbil, 44001, Iraq

Abstract

This study presents a quantitative comparison between the Jensen and Ishihara wake models to assess their impact on power output in wind farms. Both models were applied in a MATLAB-based simulation environment using pseudo-random number generation to optimize the configuration of 30 ENERCON E2 turbines on a 100-cell grid. Wind speeds between 7 m/s and 12 m/s were simulated. The Ishihara model predicted 57.96 MW at 12 m/s, corresponding to a 2.09% reduction in power, while the Jensen model predicted 55.71 MW, corresponding to a 5.89% reduction. For the Jensen model, the wake radius at x/D=10 was 4.53; for the Ishihara model, it was 2.05. The results show that the Ishihara model provides more accurate predictions of wake behavior and energy yield. At wind speeds close to the rated value (13 m/s), where wake effects become insignificant, both models agree. These results highlight the need to select an appropriate wake model in the design of wind farms to achieve optimal turbine placement and accurate energy predictions.

Keywords: Wind farm, wind turbine, Jensen wake model, Ishihara wake model, pseudo-random numbers, Enercon E2

Cite this article as: Saib, C., Zergane, S., Amroune, S., Belaadi, A., Berkache, A., & Mohamad, B. (2026). Wake effect in wind farms: Comparative study of models and their impact on power production. Journal of Thermal Engineering, 12(3), 1217–1224. https://doi.org/10.47481/jten.0024

1. Introduction

Researchers have extensively studied wind farm wake effects. Studies have examined how changes in wind speed affect energy generation, rotor loads, wake morphology, and wake evolution. Models based on the Navier-Stokes equations as well as empirical-analytical models have been created among others [1]. Among these models, analytical wake models offer several advantages over CFD models:

- Analytical models use simplified equations and, in contrast to CFD simulations, which are more demanding, especially at high resolutions—require less computing power and time.
- Unlike CFD models, which become difficult to modify, adaptable models can readily accommodate changes in wind speed, terrain, or wind turbine layout without requiring a complete model redesign.
- Analytical models depend on simplified assumptions and require less input data. CFD models require more comprehensive data.
- Speed: While CFD models might take much longer to provide correct results, they offer results quickly.
- CFD models are better for large-scale studies; therefore, analytical models are ideal for global simulations of large wind farms [2]; they are ideal for local and in-depth investigations.

Early in the 1920s, Gumilar et al. [3] and Lanchester [4] first experimented with wake modelling in their research. Their work set the stage for later studies of wake effects in wind farms and of wind turbine modeling.

*Corresponding Author
E-mail Adress: barhm.mohamad@epu.edu.iq

Submitted: 01 August 2025; **Accepted:** 19 August 2025

This paper was recommended for publication in revised form by Editorin-Chief Ahmet Selim Dalkılıç

Extending the linear momentum actuator disc concept, Houlsby et al. [5] assessed the potential power accessible to tidal turbine arrays covering the whole cross-section of an open channel. Their simplified approach removes earlier constraints on array design and on the Froude number, and accounts for the effects of far-wake mixing on overall energy recovery. According to the research, the power output of a turbine is very dependent on its area to the cross-sectional area of the channel, sometimes known as its blockage ratio. The peak power coefficient rises from 0.60 to 0.93 as the blockage ratio goes from 0.05 to 0.20. An additional 3% benefit follows a rise in the Froude number. The results indicate that turbines having smaller thrust coefficients and larger blockage ratios are more efficient because they minimize wake-mixing losses in proportion to the entire extracted power. Originally, wind wake investigations employed simple mathematical models such the Jensen model [6], which projected the downstream reduction in wind speed using a stationary methodology. Later, this model was improved by semi-empirical methods such the Larsen model [7], which more realistically represented scattering occurrences.

This study analyzes and compares two wake models: the frequently used simple, linear Jensen model [6], and a more recent Ishihara model [8], which incorporates wind turbulence. These models, along with the characteristics of the ENERCON E2 turbine [6-10], are implemented in a program developed under MATLAB to evaluate the power generated based on different wind turbine configurations. The turbine layout is optimized using pseudo-random number generation. The results of the numerical simulation are then presented and analyzed.

2. Wake models

Wind farm design primarily uses the simplified Jensen model [6] because it is simple and easy to implement. It is derived from the linear expansion coefficient of a wake. It posits that the radius gradually increases, while its influence diminishes with distance. Its precision is, however, constrained as this model does not well represent intricate events like turbulence and atmospheric boundary-layer influences.

The more comprehensive Ishihara model [8], on the other hand, develops on the Jensen model by adding these effects. In a wind farm, where all turbines are identical and located at the same height, the downstream wakes are similar for each turbine. The power generated at that time is determined by the velocity deficit and the downstream distance: a turbine outside the wake maintains its peak performance, whereas one within the wake loses power as the wind speed is reduced.

2.1. Jensen model

The simplified Jensen model [6] cannot accurately forecast wind farm power because it does not consider some important events like turbulence strength and interaction with the atmospheric boundary layer.

The wind speed within the wake, as illustrated in Figure 1 and discussed in references [11-17]:

$$U_w = U\left[1 - \sqrt{\frac{R^2\left(1 - \sqrt{1}\right)}{\left(r_w\right)^2}}\right] \tag{1}$$

The wake radius in this model at a distance x is expressed as [12-14]:

$$r_w = R + \alpha x \tag{2}$$

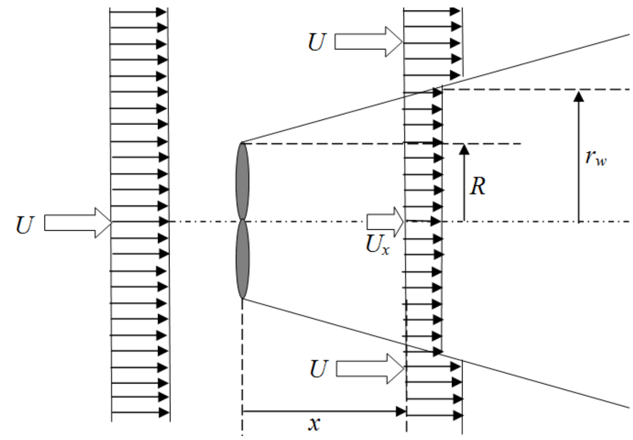

Figure 1. Physical model of the Jensen wake

3. Ishihara model

Including turbulence effects, the Ishihara model [8] builds on the Jensen model. It differentiates between ambient turbulence, caused by ground roughness and atmospheric conditions, and additional turbulence produced by an upstream wind turbine (Fig. 2). As a result, this approach enables improved prediction of downstream velocity deficit and wake growth.

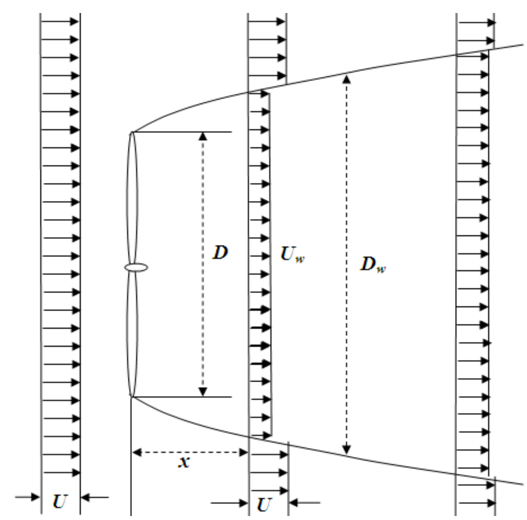

Figure 2. Physical model of the Ishihara wake

For this model, the wind speed in the wake is expressed by the equation (3)[18]:

$$U_w = U\left[1 - \frac{\sqrt{C_T}}{32}\left(\frac{1.666}{K_1}\right)^2\left(\frac{x}{D}\right)^{-p}\exp\left(\frac{r^2}{D_w^2}\right)\right] \quad (3)$$

The wake diameter is given by [18]:

$$D_w = D + \frac{K_1 C_T^{\frac{1}{4}}}{0.833} D^{1-\frac{p}{2}} x^{\frac{p}{2}} \quad (4)$$

The parameter p is a function of the ambient and added turbulence intensity, given by:

$$p = k_2(I_a + I_w) \quad (5)$$

The ambient turbulence (I_a) and the turbulence generated by the wake (I_w) are defined as follows:

$$I_a = \frac{0.5}{\ln\left(\frac{Z}{Z_0}\right)} \quad (6)$$

$$I_w = \frac{k_3 C_T}{\max(I_a, 0.03\max(I_a, 0.03)}\left(1 - \exp\left(-4\left(\frac{x}{10D}\right)^2\right)\right) \quad (7)$$

The coefficients k_1, k_2, k_3 are 0.27, 6.0, 0.004 respectively [19-21].

4. MATLAB simulation setup

A MATLAB R2024b algorithm was developed to evaluate wake effects and optimize wind turbine placement within a wind farm. The simulation assumes a uniform and unidirectional wind flow, representing the dominant wind direction across the site. Particularly in the sub-nominal region, where wake interactions are most pronounced, wind speeds between 7 m/s and 12 m/s were considered representative of the operating conditions of the ENERCON E2 turbine. The design of the wind farm was based on a grid of 100 identical square cells, each with dimensions of 10R × 10R. R denotes the length of the turbine blade. Thirty turbines were placed in the middle of the chosen cells. Wake-interaction calculations using both the Jensen and Ishihara models were performed; velocity deficits from multiple wakes were combined using the quadratic superposition approach. The simulation used pseudo-random number generation to look at different turbine layout ideas. The optimization objective was maximizing the wind farm's entire power output; Convergence was attained when the optimal setup persisted constant over multiple runs. For every layout, the effective wind speed at every turbine was computed utilizing the applied wake model; The turbine's power curve was then used to estimate the matching power output. This setup under specific boundary conditions and realistic wind conditions allowed a constant comparative evaluation of the two wake models.

4.1 Adopted wind turbine features

The simulations in this research are based on an Enercon E2 horizontal-axis wind turbine [6]; its specifications and power curve are shown in Table 1 and Figure 3.

Table 1. Features of the Enercon E2 turbine

Characteristics	Values
Rotor Diameter	82 m
Hub height	85 m
Number of blades	3
Cut-out speed	28 - 34 m/s
Rotation speed	6 - 18 rpm (variable)
Nominal power	2050 kW
Direction of rotation	Clockwise
Swept area	5281 m²
Wind speed at nominal power	13 m/s

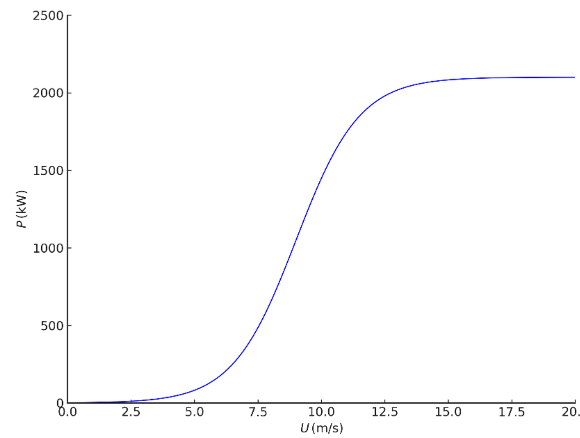

Figure 3. Power curve of the ENERCON E2 turbine [6]

Song [22] presented a two-level optimization model combining turbine placement with cable layout to maximize total profit by minimizing both cable cost and power loss, using an enhanced differential evolution and Dijkstra-based algorithm (IDEDA) in search of bettering wind farm layout optimization by tackling several aspects of the problem. Thomas et al. [23] created the Wake Expansion Continuation (WEC) technique in reaction to the complexity of multi-modality in the optimization field. WEC progressively reduces the inflated wake diameter across numerous optimizations runs, thereby enabling gradient-based approaches to avoid local minimum and to find more generally optimal layouts. Song et al. [24] focused on how to make the most of the placement of turbines and active yaw control by using particle swarm optimization (PSO). This was done while considering that the wakes were asymmetrical and the wind direction varied. This was implemented to increase the annual amount of energy produced in ways that complemented each other. Although every study focuses on a different feature, advanced wake modeling and robust optimization techniques are their common thread, as they highlight economic constraints, search-space complexity, and operational control.

5. Results and discussions

This study of the wake produced by a wind turbine, based on two different wake models, evaluates how the velocity deficit and wake expansion affect the aerodynamic performance of downstream turbines. The upstream wind speed, rotor radius, and turbine spacing primarily determine these effects.

5.1. Wind speed in the wake

Figure 4 shows how the velocity deficit in the single wake produced by the ENERCON E2 turbine changes as the wake models created by Ishihara and Jensen [6-8] assume uniform, unidirectional flow over the active disk with wind speeds ranging from 7 m/s to 12 m/s upstream of the leading wind turbine.

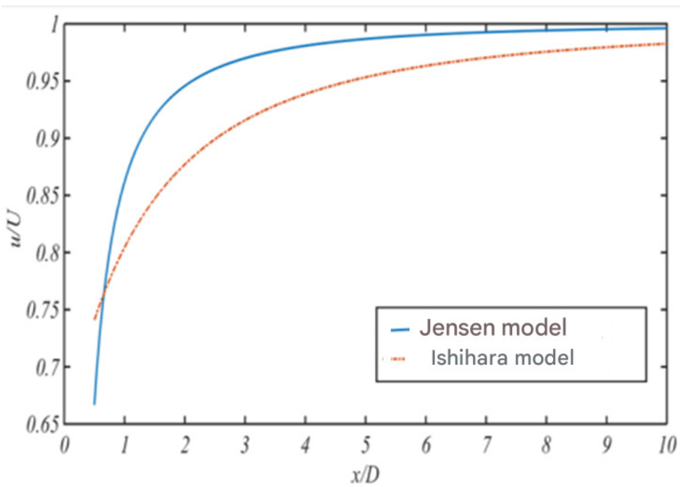

Figure 4. Speed deficit as a function of dimensionless distance x/D

The findings draw attention to these points:

- The Jensen model, in its simplified form, predicts faster recovery of speed in the wake. It also shows substantial re-acceleration as early as x/D = 0.75. Beyond x/D = 10, the two models diverge to a lesser extent, indicating a gradual weakening of the wake's effect on downstream equipment.
- The u/U ratio, which indicates how much faster the water moves in the wake than upstream, increases almost perfectly linearly with the normalized distance x/D. Immediately downstream of the rotor, the velocity deficit is highest and progressively decreases owing to turbulent mixing, asymptotically approaching its initial value.
- Beyond x/D = 10, the predictive variances of the two models are negligible, with a relative difference of roughly 1.36%, thereby verifying their convergence at large distances.
- In both analytical methods, consistent with the progressive dissipation of wake effects, the velocity field asymptotically approaches the upstream velocity as the distance from the turbine increases.

5.2. Wake radius

The wake radius is defined as the radial distance from the flow axis to the position where the velocity recovers to 99% of the upstream speed U. For velocities between 6 m/s and 11 m/s, uniform, unidirectional flow across the active disk of the ENERCON E2 turbine will result in According to the Jensen and Ishihara models [6-8], Figure 5 illustrates the evolution of the wake growth rate D_w/D with respect to the reduced distance x/D_x.

Over the entire downstream range, the Jensen model consistently predicts a larger wake diameter than the Ishihara model projects. For example, for x/D=5, the D_w/D ratio reaches 1.69 according to Ishihara and 2.77 according to Jensen; at x/D=10, these values are 2.05 and 4.53 respectively. This difference is explained by the nature of the wake expansion: linear growth in Jensen's model versus hyperbolic growth in Ishihara's model (Figure 5).

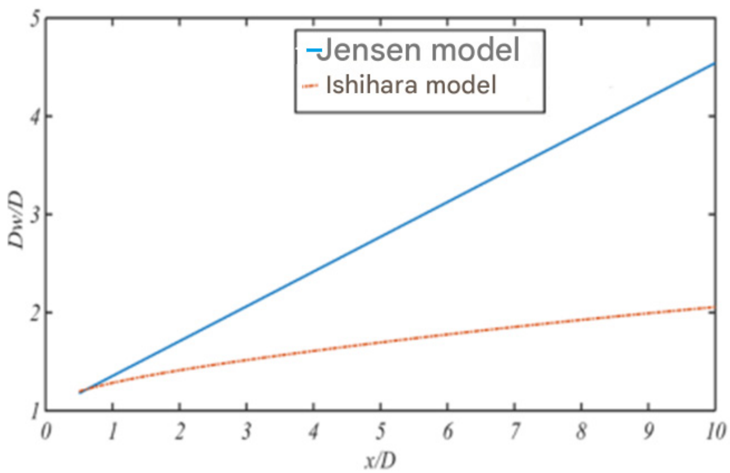

Figure 5. Wake radius rate as a function of the normalized downstream distance x/D

The proposed wind farm is subdivided into a grid of 100 uniform cells. Thirty ENERCON E2 wind turbines were chosen and were possibly positioned at the centers of these cells. Considering the wake models of Ishihara [8] and Jensen [6], the pseudo-random generation approach helps to investigate several turbine configurations. A program developed in MATLAB enables calculation of energy production for each configuration and identifies the optimal location that maximizes the power generated by all 30 turbines.

For uniform wind speeds between 7 and 12 m/s, after several iterations, the two models converge on an identical optimal configuration shown in Figure 6, where the symbols "X" indicate turbine positions relative to the dominant wind direction.

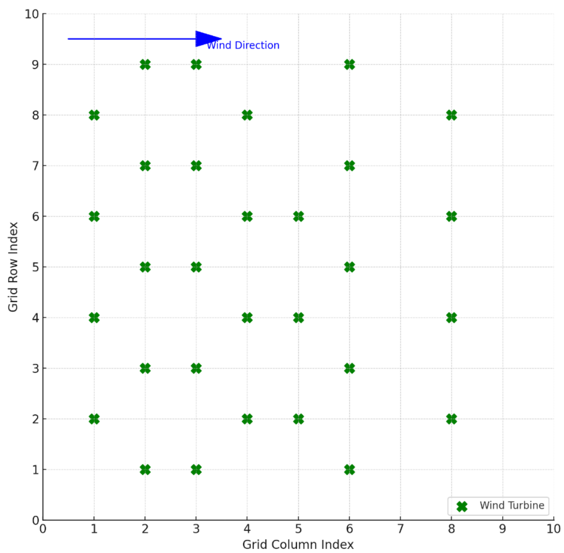

Figure 6. Optimal wind turbine layout with wind direction

The simulation results for the maximum power output and the power reduction rate of the two wake models in their optimal configurations are presented in Table 2.

Comparing the maximum power of the wind farm with and without wake effects for both models, Table 2 clearly shows that the power reduction rate, $\Delta P/P_{WWE}$, decreases with increasing wind speed. In other words, the wake effect becomes less significant as wind speed increases. At 13 m/s, corresponding to the rated wind speed of the ENERCON E2 turbine, the nominal power of the wind farm, without wake losses, is 61.5 MW. By extrapolating the trend from Table 2, the Ishihara model reaches approximately 94.2% of nominal capacity (≈57.96 MW), while the Jensen model achieves 90.6% (≈55.71 MW). These findings show that under or close to rated conditions, wake effects have a noticeably reduced impact, and the models approach the rated power restriction of the turbine.

The power curve leveling off and becoming almost constant as wind speed nears 13 m/s, which corresponds to the rated power output of 2.050 MW for the ENERCON E2 turbine (see Figure 5), helps to explain this trend. Beyond this point, the wake effect no longer influences the turbine's power output or the rate of power loss.

The detailed Ishihara model is more optimistic and better suited than the Jensen model for the following reasons:

- Higher total power.
- Lower power reduction rate.

In addition to the core findings, a sensitivity analysis was performed, as shown in Figure 7, to evaluate the influence of inter-turbine spacing on total power output. Increasing the distance between turbines by 20% led to a 6.3% and 4.7% drop in wake-induced power losses for the Jensen and Ishihara models, respectively. This means that the Jensen model is more affected by changes in spatial layout than the other model because it assumes that the wake spreads out in a straight line. The results also back up what Tian et al. [14] found earlier: that there was a 6-9% difference in overall power output between basic and advanced wake models in the same wind farm conditions. The higher predictive power of the Ishihara model is due in part to its consideration of ambient and additional turbulence effects, which accelerate wake recovery and reduce wake width, especially at speeds below the rated wind speed. These characteristics reduce overall velocity deficits and, therefore, increase energy production. Furthermore, a key design insight is the convergence of the two models at 13 m/s: wake-induced losses are minimal at the rated wind speed; therefore, the best designs found using various models could perform similarly under those circumstances. The choice of wake model is crucial for accurate prediction of energy yield in wind farms that operate predominantly in the sub-rated zone.

Table 2. Overview of maximum power output and associated power reduction rate

U(m/s)		7	8	9	10	11	12
P(MW)	Ishihara model	14.26	22.42	32.77	43.58	52.52	57.96
	Jensen model	13.56	22.11	32.66	43.23	51.32	55.71
$\Delta P/P_{WWE}$ (%)	Ishihara model	9.21	9.24	8.30	6.56	4.27	2.09
	Jensen model	13.62	10.50	8.61	7.35	6.46	5.89

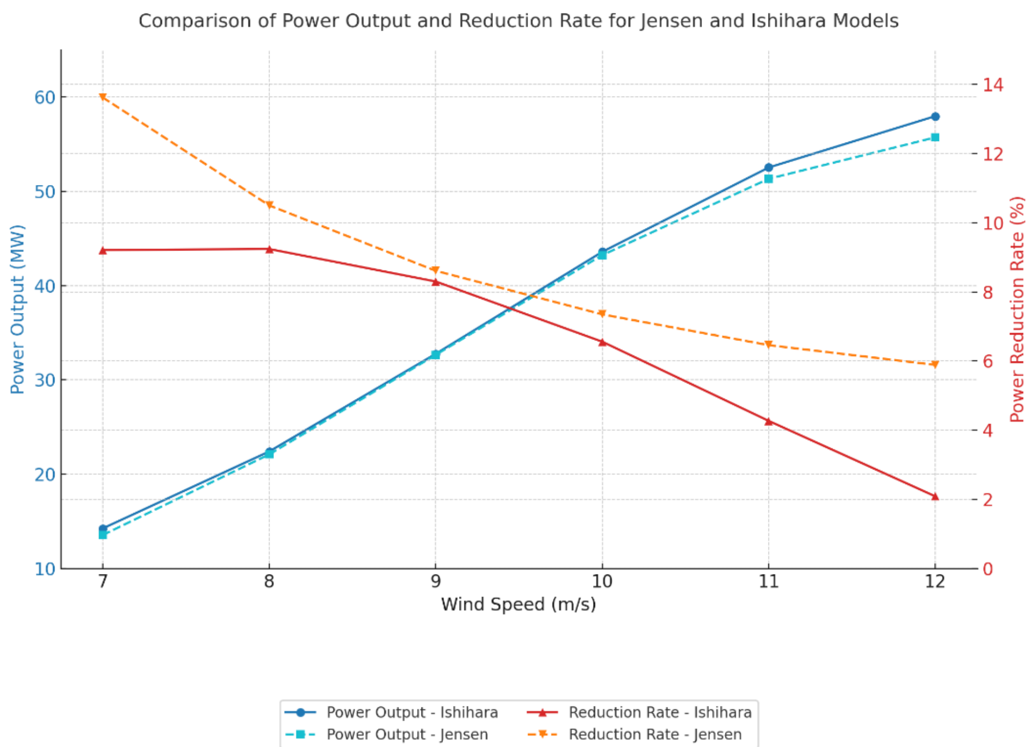

Figure 7. Comparison of power output and reduction rate for Jensen and Ishihara models

6. Conclusion

This research underlines the significant impact of wake effects on wind farm energy production through careful analysis of two reference models: Ishihara and Jensen. The results suggest that the wake effect is small beyond a certain distance because the wind speed decreases downstream and is most visible near the generating turbine.

Based on wake-width analysis, which reveals clear differences between the two models, the Jensen model predicts larger wake growth, whereas the Ishihara model predicts more moderate growth. Energy production is more efficient when using the Ishihara model; therefore, the turbine layout is directly affected.

Rising wind speed also caused the rate of power loss to fall; beyond 13 m/s — the ENERCON E2 turbine's rated speed — the loss was almost non-existent. The Ishihara model is a suitable choice for modelling wake effects in wind farms, as simulations show that it yields greater total power and a lower rate of power reduction.

Several points should be considered. Uniform, unidirectional wind flow reduces the realism of wind conditions and could have affected the convergence of both models to the same configuration. The absence of experimental validation constrains the assessment of model accuracy in practical contexts. Using pseudo-random number generation for layout optimisation might not give the best results for the whole world, but it is good for comparison. Future studies that use wind roses, employ advanced optimization methods such as genetic algorithms, and verify results with real data will enhance the reliability and utility of the findings.

Authorship contributions

Authors equally contributed to this work.

Data availability statement

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

Conflict of interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Ethics

There are no ethical issues with the publication of this manuscript.

Statement on the use of artificial intelligence

Artificial intelligence was not used in the preparation of the article.

Nomenclature

C_T	Trust coefficient
D	Turbine diameter
D_w	Wake diameter
D_x	Wake diameter at x position
I_a	Ambient turbulence intensity
I_w	Added turbulence intensity
K_1, K_2, K_3	Coefficients of Ishihara model
P	Power of wind turbine
P_{WWE}	Power without wake effects
P	Parameter
ΔP	Power reduction rate
R	Turbine radius
r_w	Wake radius
x	Wake downstream position
U	Wind speed
U_w	Downstream wind speed
Z	Hub height
Z_0	Terrain roughness
α	Wake expansion coefficient

References

[1] Neunaber, I., Hölling, M., & Obligado M. Leading effect for wind turbine wake models. Renew Energy. 2024;223:119935. https://doi.org/10.1016/j.renene.2023.119935

[2] Vogel, C. R., & Willden, R. H. Investigation of wind turbine wake superposition models using Reynolds-averaged Navier-Stokes simulations. Wind Energy, 2020; 23(3),593-607. https://doi.org/10.1002/we.2444

[3] Gumilar, L.; Sholeh, M.; Triharto, R.; Rumokoy, S.N.; Monika, D.; Aji, A.F. Influence of Wind Turbine Pitch Angle on DFIG Output Stability under Load Changes. In Proceedings of the 2021 4th International Seminar on Research of Information Technology and Intelligent Systems(ISRITI),Yogyakarta, Indonisia, 16 December 2021; IEEE: Piscataway, NJ, USA, 2021; pp. 435–439. https://doi.org/10.1109/ISRITI54043.2021.9702760

[4] Lanchester, F.W. A contribution to the theory of propulsion and the screw propeller. J. Am. Soc. Nav. Eng. 1915; 27, 509–510. https://doi.org/10.1111/j.1559-3584.1915.tb00408.x

[5] Houlsby, G. T., & Vogel, C. R. (The power available to tidal turbines in an open channel flow. Proceedings of the Institution of Civil Engineers-Energy, 2017; 170(1), 12-21. https://doi.org/10.1680/jener.15.00035

[6] Jensen, N. O. A note on wind generator interaction. Roskilde: Risø National Laboratory, 1983; Report No.: Risø-M-2411.

[7] Larsen, G. C. A simple stationary semi-analytical wake model. Risø National Laboratory for Sustainable Energy, Technical University of Denmark, Denmark. Forskningscenter Risoe, 2009; Risoe-R No. 1713.

[8] Ishihara, T. Qian, G-W. A new Gaussian-based analytical wake model for wind turbines considering ambient turbulence intensities and thrust coefficient effects. Journal of Wind Engineering and Industrial Aerodynamics, 2018;177:275–92. https://doi.org/10.1016/j.jweia.2018.04.010

[9] Zergane, S., Farsi, C., Amroune, S., Benkherbache, S., & Menasri, N. A New Study on the Effect of the Partial Wake Generated in a Wind Farm. Energies, 2024; 17(6), 1498. https://doi.org/10.3390/en17061498

[10] Li, L., Wang, Y. M., & Liu, Y. Q. (2013, September). Impact of wake effect on wind power prediction. In 2nd IET Renewable Power Generation Conference, RPG 2013; (pp. 3-C20). Stevenage UK: IET. https://doi.org/10.1049/cp.2013.1827

[11] Croonenbroeck, C., & Hennecke, D. Advances in Wind Farm Layout Optimization: Wind Direction Robustness and Wake Induced Asymmetric Thrust Load. Journal of Energy and Power Technology, 2021; 3(4), 1-28. https://doi.org/10.21926/jept.2104044

[12] Zergane, S., Amroune, S., Rokbi, M., & Guesmia, S. New study on the extension of a current wind farm, case of Kaberten park in Algeria. Turk. J. Computer Math. Educ, 2022; 13, 428-434. https://doi.org/10.17762/turcomat.v13i2.12281

[13] Akan, A. P., & Akan, A. E. (2021). Determination of Tekirdağ (Turkey) wind energy potential and evaluation of selected wind turbines in terms of techno-economic and environmental aspects. Journal of Renewable and Sustainable Energy, 2021; 13(3). https://doi.org/10.1063/5.0053871

[14] Tian, L., Zhu, W., Shen, W., Song, Y., & Zhao, N. (2017). Prediction of multi-wake problems using an improved Jensen wake model. Renewable energy, 2017; 102, 457-469. https://doi.org/10.1016/j.renene.2016.10.065

[15] Marmidis, G., Lazarou, S., & Pyrgioti, E. Optimal placement of wind turbines in a wind park using Monte Carlo simulation. Renewable energy, 2005; 33(7), 1455-1460. https://doi.org/10.1016/j.renene.2007.09.004

[16] Grady, S.A., Hussaini, M.Y., Abdullah, M.M. Placement of wind turbines using genetic algorithms, Renewable Energy, 2005; vol. 30, pp. 259-270. https://doi.org/10.1016/j.renene.2004.05.007

[17] Moseti, G., Poloni, C., Diviacco, B. Optimization of wind turbine positioning in large wind farm by means of a genetic algorithms, Journal of Wind Engineering and Industrial Aerodynamics, 1995; Issue 51, pp 105-116. https://doi.org/10.1016/0167-6105(94)90080-9

[18] Tong, W., Chowdhury, S., Zhang, J., & Messac, A. Impact of different wake models on the estimation of wind farm power generation. In 12th AIAA aviation technology, integration, and operations (ATIO) conference and 14th AIAA/ISSMO multidisciplinary analysis and optimization conference, 2012; p. 5430. https://doi.org/10.2514/6.2012-5430

[19] Charhouni, N., Arbaoui, A., & Sallaou, M. Analysis of wake impact on wind farm performance using two analytical models. In 2014 International Renewable and Sustainable Energy Conference (IRSEC), 2014; October; (pp. 323-327). Ieee. https://doi.org/10.1109/IRSEC.2014.7059780

[20] Charhouni, N. Qualification of three analytical wake models. In CFM 2015-22ème Congrès Français de Mécanique. AFM, Maison de la Mécanique, 2015, August;. 39/41 rue Louis Blanc-92400 Courbevoie.

[21] Alabi, O. O., Adeaga, O. A., & Dare, A. A. Reviewing the enhancement optimum performance characteristics of horizontal axis wind turbine blades using add-on of suitable aerodynamic properties, Journal of Thermal Engineering, 2024; 10(4):1092-1106. https://doi.org/10.14744/thermal.0000848

[22] Song E. A bi-level optimization model and improved algorithm for wind farm layout. IET Renew Power Gener. 2024;18:2017–2033. https://doi.org/10.1049/rpg2.13005

[23] Thomas, J. J., McOmber, S., & Ning, A. Wake expansion continuation: Multi-modality reduction in the wind farm layout optimization problem. Wind Energy. 2022; 25(4): 678-699. https://doi.org/10.1002/we.2692

[24] Song, J., Kim, T., & You, D. Particle swarm optimization of a wind farm layout with active control of turbine yaws. Renewable Energy. 2023;206:738–747. https://doi.org/10.1016/j.renene.2023.02.058