



## Research Article

# A novel modified JAYA algorithm for heat exchanger optimization

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## ARTICLE INFO

### Article history

Received: 18 July 2023

Revised: 17 November 2023

Accepted: 19 November 2023

### Keywords:

Grey Wolf Optimizer; Heat Exchanger Optimization; JAYA Algorithm; Modified JAYA Algorithm; Sine Cosine Algorithms

## ABSTRACT

In general, algorithm modification is changing or alternating some aspects of the original algorithms with improving their performances. This work aims to introduce and implement a novel modified Jaya algorithm (MJ) to optimize fins and tube heat exchangers. The objective functions used in the current work are to minimize total cost and maximize effectiveness. The optimization results of the MJ were compared with the standard JAYA algorithm and another two different algorithms, namely the Grey Wolf Optimizer (GWO) and Sine Cosine Algorithms (SCA), to examine the MJ performance improvement. A MATLAB in-house code was used to obtain the results of the different optimizing algorithms. Each of the four algorithms optimized the heat exchanger at three different values of population size, which are 25, 50, and 100, and three different numbers of runs, 20, 40, and 80, to determine the optimal solution. The results showed that MJ outperforms the standard JAYA algorithm and SCA in all cases studied. MJ performs better than GWO at low and medium populations, 25 and 50. Still, at a population size of 100, MJ and GWO perform equally, with the advantage that MJ obtains less average execution time to find optimal solutions than GWO. The time increase of GWO over MJ is 450.56% at maximum and 52.86% at minimum.

**Cite this article as:** Ahmed A, Elmardi O, Elmahi F, Younis O, Abdelrahman M. A novel modified JAYA algorithm for heat exchanger optimization. J Ther Eng 2024;10(4):986–1010.

## INTRODUCTION

The desire to design and manufacture more efficient energy systems considering the cost-effective importance makes optimizing energy systems very popular nowadays [1]. Fins and tube heat exchangers (FTHE) are regarded as

one of the energy systems used to transfer thermal energy between liquids and gases [2]. Depending on the purpose of the applications, fins can be set inside, outside, and both inside and outside the tubes [3]. The Objective functions used to optimize FTHE can be generalized into minimizing cost, maximizing effectiveness, minimizing weight,

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This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkılıç



maximizing heat transfer process, minimizing pressure drop, and so on [4]. Using Advanced optimization algorithms in the heat exchangers optimization process makes it easier and more accurate [5]. For that, several researchers put their efforts into implementing the newly invented optimization algorithms in heat exchanger optimization and evaluating their performance, in addition to the research concerned with hybridizing algorithms and modifying existing algorithms to enhance their performance. Figure 1 illustrates the schematic configuration of the FTHE and its terminology. Regarding the FTHE shown in Figure 1, the fins are set outside the tubes, and the tubes are arranged in a staggered arrangement. Practically, the liquid fluids pass through inside the tubes and the gaseous fluid outside the tubes through the fins.

Yang et al. [6] optimized the design parameters for an FTHE of a residential refrigerator under frost conditions to enhance its thermal performance. They examined the average rate of heat transfer utilizing ice mass and operating time as objective functions, and reaction surface and Taguchi techniques were used as optimization tools. Seven design parameters were considered in Xie et al. [7,8] to minimize the weight and yearly cost of FTHE optimization employing conventional thermal design and genetic algorithm GA. FTHE in a refrigerant circuit has been optimized in [9] using GA. The objective function used was tube length minimization and heat transfer maximization. In [10], authors fabricated a mathematical model identifying the FTHE condenser optimal dimensions of the HVAC system; the objective function adapted is entropy generation. A comprehensive economic and thermal optimization of FTHE was conducted in [11,12]. In these works, authors used effectiveness minimization and annual cost as objective functions to optimize the heat exchanger employing GA and NSGA-II. An optimization using the Taguchi method associated with the parametric study of louver FTHE has been performed by Hsieh and Jang [13]. Their results stated that the thermohydraulic performance of the louver FTHE is highly affected by fin pitch, fin collar outside diameter, and transverse tube pitch. The latest thermohydraulic model of FTHE available in the literature has been established in [14] based on the correlations listed in Wang et al. works [15-18]. In this work, authors optimized FTHE using a heat transfer search algorithm HTS based on minimizing weight and total cost. A combined optimization using NSGA-II and CFD analysis of FTHE with ellipticity tubes had been performed by Zhang et al. [19]. The results show that at Reynold's number of 541 using tube ellipticity of 0.34, the pressure drop of the FTHE decreased by 20%. By using the adaptive multi-objective differential evolution algorithm, Yuan et al. [20] performed a multi-objective optimization of micro-fins helically coiled tube heat exchangers considering the total cost and the effectiveness as objective functions. Liu et al. [21] adapted the topology optimization to study the thermal and hydraulic performance of the FTHE. According to the findings, the FTHE's

improved construction increased thermal performance by 22.64 % to 28.04 % and decreased hydraulic performance by 33.37 % to 47.72 %. Modelling and optimization using GA of a branch type FTHE had been performed by Dehaj et al. [22]. In this study, authors took into consideration eight design variables and adopted the total cost besides the effectiveness as an objective function. The results indicate that the optimized branch type FTHE achieved an improvement of 6.5% in effectiveness maximization compared to the conventional FTHE. Design optimization using Continuous GA and PSO of high-temperature FTHE manifold associated with a CFD-based simulation using ANSYS CFX software had been conducted in [23]. In this work, the new optimized FTHE shows a reduction of the tube wall temperature from 185 °C to 134 °C in addition to almost five times compressible stress reduction compared to the conventional FTHE.

Based on the previously listed literature, the researchers observed that most works adapted classical algorithms and methods to optimize FTHE. At the same time, in the last few years, several new optimization algorithms have been

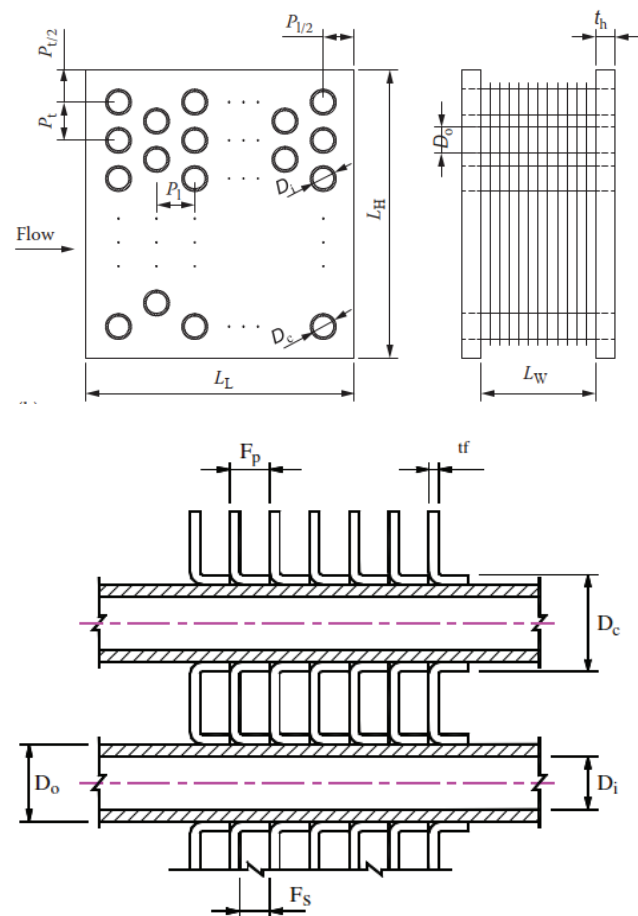


Figure 1. Schematic configuration of the FTHE and its terminology.

invented. That newly invented optimization algorithm has not been implemented yet for optimizing heat exchangers and examining the superiority over the classical algorithms for that task. JAYA algorithm is one of the simplest population-based optimization algorithms introduced in the last decade. Moreover, the JAYA algorithm is parameterless since it requires no specific control parameters besides the standard parameters that algorithms usually require. Zitar et al. [24] conducted an excessive and comprehensive review of the JAYA algorithm and its modified versions associated with their applications. Gholami et al. [25] introduced an improved powerful version of the JAYA algorithm for efficient optimization of engineering problems. In the field of heat exchanger optimization, Zhang et al. [26] invented A new dynamic opposite learning that improved Jaya and applied it in the design optimization of the plate-fin heat exchanger as an industrial benchmark. Due to the simplicity of its updating position equation and the parameterless advantage, JAYA attracts the attention of researchers in this article.

The current work contributions can be summarized as follows:

To introduce a novel modified JAYA algorithm MJ to enhance performance compared to the standard JAYA. Moreover, applying MJ to optimize FTHE using minimum annual cost and maximum practical effectiveness as an objective function. The optimization results of MJ will be compared with the standard JAYA results and two different optimization algorithms, precisely the Grey Wolf Optimizer (GWO) and Sine Cosine algorithm (SCA), to assess the superiority of different algorithms used over each other. The four algorithms were implemented using MATLAB in-house code.

In contrast to prior publications, the authors of this research employed three alternative population size values (25, 50, and 100) and three numbers of runs (20, 40, and 80) instead of a single value to determine the dominance and superiority of a specific algorithm over the algorithms previously listed.

**OPTIMIZATION**

This section briefly describes standard JAYA and the modifications conducted to enhance its performance, objective function, subjected constraints, design variables, and case study data used.

**JAYA Algorithm**

The standard JAYA algorithm was invented first by Rao [27]. This algorithm belongs to the population-based optimization algorithms classification. As one of the population-based strategies, JAYA begins the optimization procedure with a collection of random solutions. A particular objective function frequently assesses that randomly generated solution to reach the optimal values. The position

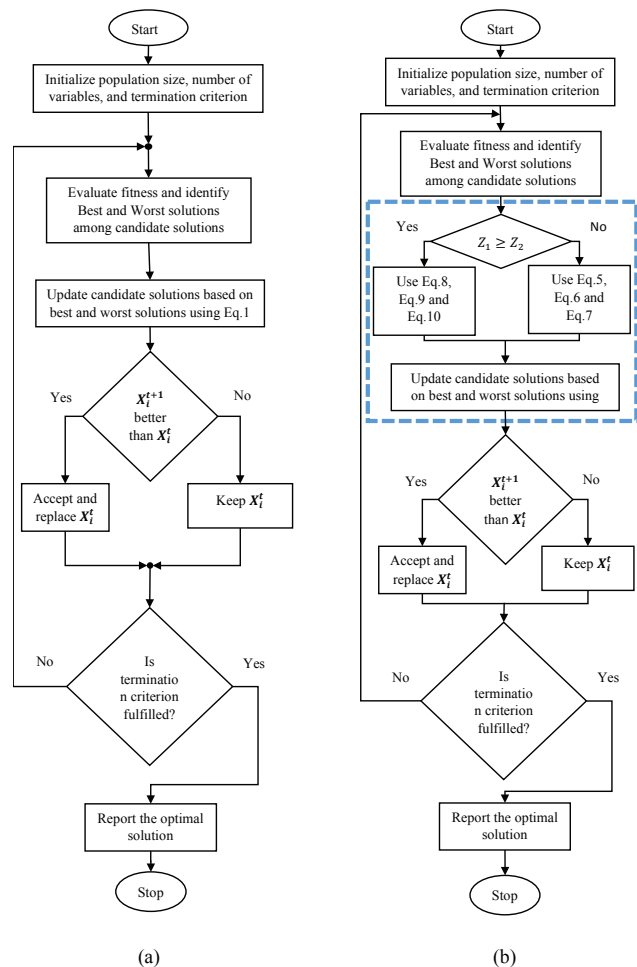
updating equation for the standard JAYA algorithm can be written as in Eq.1 shown below [27]:

$$X_i^{t+1} = X_i^t + r_1(X_{Best} - |X_i^t|) - r_2(X_{Worst} - |X_i^t|) \quad (1)$$

In the above equation,  $X_i^t$  denotes the immediate solution at the particular dimension ( $i$ ) and specific iteration ( $t$ ).  $X_{worst}$  represents the worst solution whereas  $X_{Best}$  the best candidate solution. The  $r_1$  and  $r_2$  both are randomly generated numbers in the range of [0,1]. Figure 2 (a, b) illustrates the standard and modified JAYA flow charts.

**Modified JAYA Algorithm**

The modifications of the standard JAYA algorithm introduced in this article mainly focus on two points. The first modification point is to control the random numbers,  $r_1$  and  $r_2$ , generation. In the standard JAYA, both numbers are generated randomly between 0 and 1, regardless of the values of differences  $(X_{Best} - |X_i^t|)$  and  $(X_{worst} - |X_i^t|)$ , where  $(X_{Best} - |X_i^t|)$  represent the tendency to reach the



**Figure 2.** Flow charts of Standard JAYA (a) and Modified JAYA (b).

best solution and  $(X_{worst} - |X_i^t|)$  the tendency to move away from the worst solution. The suggested modification is to control the values of  $r_1$  and  $r_2$  based on the average absolute values of the differences  $(X_{Best} - |X_i^t|)$  and  $(X_{worst} - |X_i^t|)$ . If the average value of  $|(X_{Best} - |X_i^t|)|$  is greater than or equal to the average value of  $|(X_{worst} - |X_i^t|)|$  then  $r_1$  will be a random number in a range of  $[0.5,1]$  and  $r_2$  will have a range of  $[0,0.5]$ , and if the average value of  $|(X_{Best} - |X_i^t|)|$  is less than the average value of  $|(X_{worst} - |X_i^t|)|$  then  $r_1$  in the range of  $[0,0.5]$  and  $r_2$  in the range of  $[0.5,1]$ . The second modification point is inserting an additional auxiliary weight to the position updating equation Eq.1 mentioned above. Mathematically, these modifications can be written as follows:

$$X_i^{t+1} = X_i^t + r_1(X_{Best} - |X_i^t|) - r_2(X_{Worst} - |X_i^t|) - r_3(X_{Worst} - |X_i^t|) \quad (2)$$

Let  $Z_1$  and  $Z_2$  represent the average absolute differences.

$$Z_1 = \frac{\sum|(X_{Best}-|X_i^t|)|}{D} \quad (3)$$

$$Z_2 = \frac{\sum|(X_{Worst}-|X_i^t|)|}{D} \quad (4)$$

$D$  denotes the optimization problem dimension (number of design variables).

$$\text{if } Z_1 \geq Z_2$$

$$r_1 = (0.5 \times rand) + 0.5 \quad (5)$$

$$r_2 = 0.5 \times rand \quad (6)$$

$$r_3 = |r_1 - r_2|/r_1 \quad (7)$$

$$\text{if } Z_1 < Z_2$$

$$r_1 = 0.5 \times rand \quad (8)$$

$$r_2 = (0.5 \times rand) + 0.5 \quad (9)$$

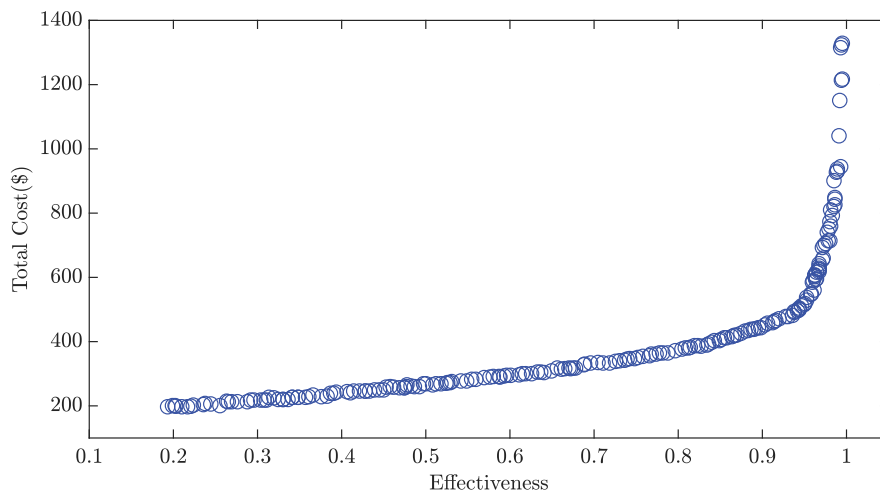
$$r_3 = |r_1 - r_2|/r_2 \quad (10)$$

The hypothesis is that applying the abovementioned modifications will enhance the MJ performance compared to the standard JAYA. For further details regarding SCA refer to reference [28] and more description about GWO is available in [29].

**Objective Function**

In this article, the objective function considered was minimizing annual cost ( $C_{tot}$ ) and maximizing effectiveness ( $\epsilon$ ) which are the main factors that lead to the design of more efficient energy systems considering the cost-effective. The aims of the optimization process of the FTHE considering the thermodynamic economic objectives are usually to minimize the total cost and to maximize the heat exchanger’s effectiveness. Cost minimization and effectiveness maximization are two conflicting objectives. This conflict is shown in Figure 3.

The annual cost is calculated based on [14] using Eq.11 to Eq. 13 as follows:



**Figure 3.** Pareto distribution of the optimal solutions points.

$$C_{tot} = C_{in} + C_{op} \quad (11)$$

$$C_{in} = C_A A^n \quad (12)$$

$$C_{op} = \left( \frac{k_{el} \tau \Delta P_a V_{t,a}}{\eta} \right) + \left( \frac{k_{el} \tau \Delta P_w V_{t,w}}{\eta} \right) \quad (13)$$

$C_{in}$  and  $C_{op}$  denote initial and operating costs.  $A$  and  $C_A$  represent the total surface area and the price of a unit of the surface area, respectively.  $k_{el}$  is the price of the electricity, whereas  $V_t$  denotes the volumetric flow rate. Pump/compressor efficiency is  $\eta$  whereas  $\tau$  represents operation hours. The pressure drop in the air side (fins side) is represented by  $\Delta P_a$ , while pressure drop in the water side (tubes side) is represented by  $\Delta P_w$ . The details and mathematical formulation to calculate  $\Delta P_a$  and  $\Delta P_w$  is available in [15-17].

The FTHE type belongs to crossflow heat exchangers, whereas both fluids are unmixed. The effectiveness ( $\epsilon$ ) of this type is calculated as Eq.14, listed below based on [2,3].

$$\epsilon = 1 - \exp \left( \frac{NTU^{0.22}}{C} (\exp(-C * NTU^{0.78}) - 1) \right) \quad (14)$$

$C$  and  $NTU$  are the heat capacity ratio and the number of transfer units calculated as in Eq.15 and Eq.16 [2,3].

$$C = \frac{c_{min}}{c_{max}} = \frac{(\dot{m}C_p)_{min}}{(\dot{m}C_p)_{max}} \quad (15)$$

$$NTU = \frac{U_o A}{c_{min}} \quad (16)$$

$U_o$  represents the overall heat transfer coefficient.

To handle the multi-objective optimization epsilon constraints method is adapted. Based on that technique, the total annual cost is seated as the primary objective function, and effectiveness is seated as the secondary objective function.

### Applied Constraints

The set of constraints applied to the earlier-mentioned objective function can be summarized as follows:

The maximum allowable pressure drop in air and water sides ( $\Delta P_{a,max}$ ,  $\Delta P_{w,max}$ ) was considered as 3% of the entrance pressure for each [2]. Based on this approximation, it will have values of 7.5 kN/m<sup>2</sup> and 6 kN/m<sup>2</sup>, respectively.

$$\Delta P_a \leq 7.5 \text{ kN/m}^2 \quad (17)$$

$$\Delta P_w \leq 6 \text{ kN/m}^2 \quad (18)$$

The ratio of the tube's Length used to its outer diameter ( $\frac{L}{D_o}$ ) must be greater than or equal to 60 [2].

$$\frac{L}{D_o} \geq 60 \quad (19)$$

$Re_w$  and  $Re_a$  denote Reynolds number for water and air sides and its ranges are as follows [15]:

$$2300 \leq Re_w \leq 2 \times 10^6 \quad (20)$$

$$300 \leq Re_a \leq 20000 \quad (21)$$

Generally, the main purpose of using fins in FTHE is to enhance the heat transfer area. In practice, the ratio of the total area of the FTHE ( $A$ ) to its fins area ( $A_f$ ) must range between 1 and 1.2 in value, as shown in Eq. 22 [4,14].

$$1 \leq \frac{A}{A_f} \leq 1.2 \quad (22)$$

Reference [11] stated that the practical effectiveness of the FTHE is in the range of 0.5 to 0.78 ( $0.5 < \epsilon < 0.78$ ). Based on that, the maximum practical effectiveness ( $\epsilon_{max}$ ) is limited to 0.78.

### Design Variables

This article considered seven design variables which are transverse and longitudinal pitches ( $P_t, P_l$ ), fin pitch  $F_p$ , the cold flow and the No-flow paths length ( $L_w, L_H$ ), outer tube diameter  $D_o$ , and the thickness of the fins  $t_f$ . The considered design variables and their upper and lower limits are shown in Table 1. The ratio of the tube's inner to outer diameters is 0.8.

**Table 1.** Design variables considered and their limits

Design Variables (mm)	Lower Limit	Upper Limit
$P_l$	12.7	32
$P_t$	20.4	31.8
$F_p$	1	8.7
$D_o$	6.8	12.7
$L_w$	200	1000
$L_H$	200	1000
$t_f$	0.1	0.33

**Table 2.** Case study data from [3,11]

Conditions	Air	Water
The flow rate in Kg/s, $\dot{m}$	2.5	3.2
Inlet temperature in °C, $T_i$	152	12
Outlet temperature in °C, $T_o$	40	-
Inlet pressure in Kpa, $P_i$	250	200
Electricity price in \$/MWh, $k_{el}$	25	25
The exponent of nonlinear, n	0.6	-
Operating time in h/year, $\tau$	5000	5000
Efficiency, $\eta$	0.65	0.65
Cost of construction materials in \$/m <sup>2</sup> , $C_A$	85	-

**Table 3.** The cases and their specific population sizes and number of runs

Cases No.	Abbreviations	Number of Runs	Population Sizes
1	R20P25	20	25
2	R20P50	20	50
3	R20P100	20	100
4	R40P25	40	25
5	R40P50	40	50
6	R40P100	40	100
7	R80P25	80	25
8	R80P50	80	50
9	R80P100	80	100

**Case Study Data**

The FTHE case study data used to test the algorithm introduced in this paper, shown in Table 2, is collected based on [3,11]. Although some of the case study data particularly electricity price and cost of construction materials are changed dramatically in recent years in the current work the same values will be used to provide a consistent base to validate the results in the current work particularly the results of the economic objective function.

The properties of both fluids (air and water) used to conduct this paper were collected from the thermophysical properties tables of Rogers and Mayhew [30].

**RESULTS AND DISCUSSION**

In this article, optimization of FTHE was conducted using the novel modified JAYA algorithm MJ introduced before in addition to three other different algorithms, namely JAYA, SCA, and GWO. Nine tested cases using three alternative population sizes (25, 50, and 100) and three different numbers of runs (20, 40, and 80) are examined, as shown in Table 3. Termination criteria for all algorithms used in the paper were 5000 iterations.

Before plunging into the statistical analysis of each algorithm used, initially must inspect if significant differences exist between their performance. This inspection could be made by applying the Friedman test (F-test). The F-test procedures start by considering two hypotheses the null hypothesis ( $H_0$ ) and the alternative hypothesis ( $H_1$ ). The hypothesis ( $H_0$ ) is that there are no differences between the algorithms' performance, whereas hypothesis ( $H_1$ ) is that at least one algorithm tends to perform better than the rest. The criteria that lead to deciding to accept or reject one of the two hypotheses is to compare the calculated test statistics ( $T_2$ ) and the tabulated statistics ( $F_{1-\alpha,k-1,(b-1)(k-1)}$ ) from the F-test tables at a particular significance level ( $\alpha$ ). In the tabulated statistics ( $F_{1-\alpha,k-1,(b-1)(k-1)}$ ),  $\alpha$  is the significance level, whereas  $b$  and  $k$  are the number of runs and the number of compared data groups. In this study significance level of  $\alpha$  equal to 0.05, which means results in a confidence level

of 95%, was used to perform the F-test considering four data groups (MJ, JAYA, SCA, and GWO). Table compares the calculated and tabulated statistics for the different cases examined. Further details regarding the Friedman test and F-test tables are existing in [31] and the F-test tables used are available in [32].

Box and whisker plot, or simply box plot, is one of the visualization methods in descriptive statistics. This method is used to demonstrate and compare numerical data groups. The comparisons of the data groups take place based on seven main comparing points, which are minimum (Min), maximum (Max), median, upper quartile (UQ), interquartile range (IQR), lower quartile (LQ), that illustrates data spread, and graph shift (higher or lower). This technique will be applied to compare the four algorithms implemented results, as Simon recommended in [33].

Figures starting from Figure 4 to Figure 12 illustrate the box and whisker plots for all examined cases. Based on Figure 4 it is found that MJ obtained Min, Max, UQ, LQ, IQR, and a median of 319.8741, 336.8258, 319.8741, 319.8741, 0, and 319.8741, respectively, while GWO obtained 320.0120, 321.0708, 320.5348, 320.181, 0.3538, and 320.3508, SCA obtained 320.8863, 323.1969, 322.4299, 321.3187, 1.1112, and 321.8308 and JAYA obtained 320.1549, 402.4073, 339.331,

**Table 4.** Comparisons of F-test statistics

Cases	Calculated statistics $T_2$	$(F_{1-\alpha,k-1,(b-1)(k-1)})$
R20P25	66.20	2.77
R20P50	7.80	2.77
R20P100	4.74	2.77
R40P25	127.13	2.68
R40P50	23.90	2.68
R40P100	11.97	2.68
R80P25	169.58	2.64
R80P50	36.92	2.64
R80P100	21.57	2.64

321.8723, 17.4587, and 329.9196. The Min, Max, UQ, LQ, IQR, and median of MJ in Figure 5 are 319.8741, 372.7102, 324.7113, 319.8741, 4.8372, and 319.8741, while are 319.9346, 321.0254, 320.4974, 320.0397, 0.4577, and 320.1746 for GWO, and 320.4974, 322.3431, 321.7377, 321.0952, 0.6425, and 321.3049 for SCA and are 319.8741, 462.9882, 340.2568, 319.9849, 20.2719, and 321.0463 for JAYA. In Figure 6, the Min, Max, UQ, LQ, IQR, and median values are 319.8741, 436.9370, 329.7285, 319.8741, 9.8544, and 319.8741 for MJ, respectively, and 319.9050, 321.0438, 320.3488, 319.9784, 0.3704, and 320.0893 for GWO, and 320.5393, 321.7573, 321.5064, 321.0376, 0.4688, and 321.3559 for SCA and 319.8741, 435.2783, 363.5612, 320.0596, 43.5016, and 323.3264 for JAYA. Figures 4, 5, and 6 below show that MJ achieved the lowest UQ, LQ, IQR, and median values compared to JAYA; furthermore, MJ graphs are slightly lower than JAYA graphs. The above discussions indicate that MJ performs better than JAYA through cases R20P25, R20P50, and R20P100.

For case R40P25, Figure 7, MJ achieved Min, Max, UQ, LQ, IQR, and a median of 319.8741, 430.1641, 319.8741, 319.8741, 0, and 319.8741, respectively, whereas GWO achieved 319.9630, 321.5775, 320.5483, 320.181, 0.3673, and 320.3909, and SCA obtained 320.8863, 323.1969, 322.4299, 321.3187, 1.1112, and 321.8308 and JAYA obtained 319.8751, 424.6662, 357.8509, 322.4198, 35.4311, and 333.4499. In Figure 8, case R40P50, the Min, Max, UQ, LQ, IQR, and median values are 319.8741, 374.1942, 319.8741, 319.8741, 0, and 319.8741 for MJ, respectively, whereas 319.9346, 321.0254, 320.3609, 320.038, 0.3229, and 320.158 for GWO, and 320.4974, 322.9174, 321.8351, 321.2301, 0.605, and 321.4853 for SCA and 319.8741, 462.9882, 338.6432, 319.9789, 18.6643, and 321.219 for JAYA. Figure 9, case R40P100, found that MJ achieved Min, Max, UQ, LQ, IQR, and a median of 319.8741, 436.9370, 328.0736, 319.8741, 8.1995, and 319.8741, respectively,

whereas GWO achieved 319.9050, 321.0438, 320.3098, 319.9761, 0.3337, and 320.0384, and SCA obtained 320.3375, 322.2990, 321.5941, 320.9654, 0.6287, and 321.3214 and JAYA achieved 319.8741, 435.2783, 351.2601, 319.8784, 31.3817, and 322.0495. From the above-presented discussions, MJ supreme compared to JAYA through cases R40P25, R40P50, and R40P100 since it obtained UQ, LQ, IOR, and median values less than JAYA. In addition to that, MJ graphs are slightly lower than JAYA graphs.

In the case of R80P25, shown in Figure 10, MJ records Min, Max, UQ, LQ, IQR, and a median of 319.8741, 430.1641, 319.8741, 319.8741, 0, and 319.8741, whereas GWO records 319.9569, 321.5775, 320.5517, 320.1261, 0.4256, and 320.366, and SCA obtains 320.7090, 323.2727, 322.2951, 321.5425, 0.7526, and 321.8643 while JAYA achieves 319.8751, 477.9597, 358.0492, 323.6657, 34.3835, and 333.7584. In Figure 11, case R80P50, the Min, Max, UQ, LQ, IQR, and median values are 319.8741, 440.7007, 319.8741, 319.8741, 0, and 319.8741 for MJ, respectively, and 319.9346, 321.2417, 320.3609, 320.0396, 0.3213, and 320.1417 for GWO, and 320.4974, 323.0380, 321.8229, 321.2387, 0.5842, and 321.5515 for SCA and 319.8741, 468.7314, 345.877, 320.0134, 25.8636, and 321.8052 for JAYA. For case R80P100 illustrated in Figure 12, MJ has a Min, Max, UQ, LQ, IQR, and a median of 319.8741, 457.2318, 320.2911, 319.8741, 0.417, and 319.8741, respectively, while GWO has 319.9050, 321.0438, 320.2339, 319.9727, 0.2612, and 320.0211, and SCA has 320.3375, 322.2990, 321.5215, 320.9678, 0.5537, and 321.2862, whereas JAYA obtains values of 319.8741, 435.2783, 352.8838, 319.8774, 33.0064, and 320.7374. The observation is that, for cases shown in Figures 10, 11, and 12, MJ has the lowest UQ, LQ, IOR, and median values; furthermore, MJ graphs are slightly lower than JAYA graphs, which leads to saying MJ has superior performance compared to JAYA.

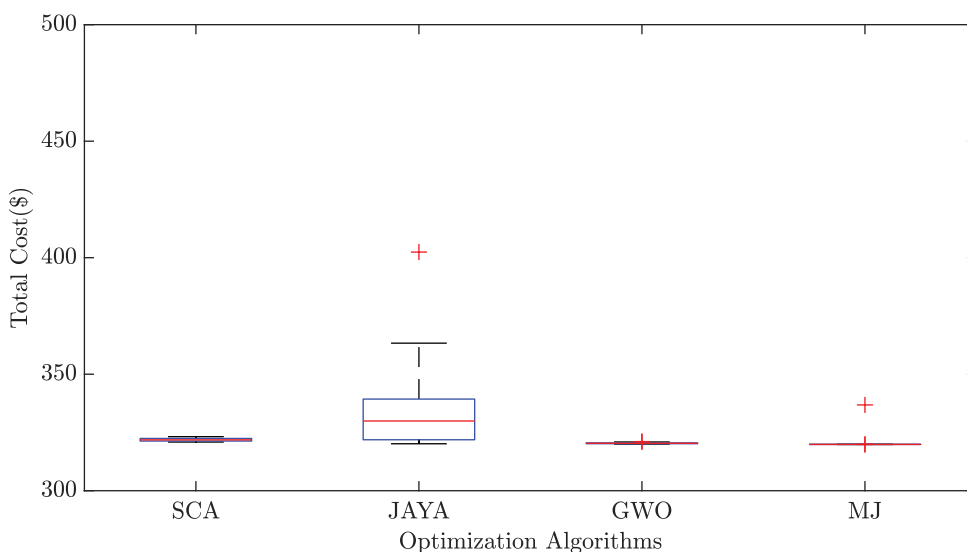


Figure 4. Box and whisker plot for the case R20P25.

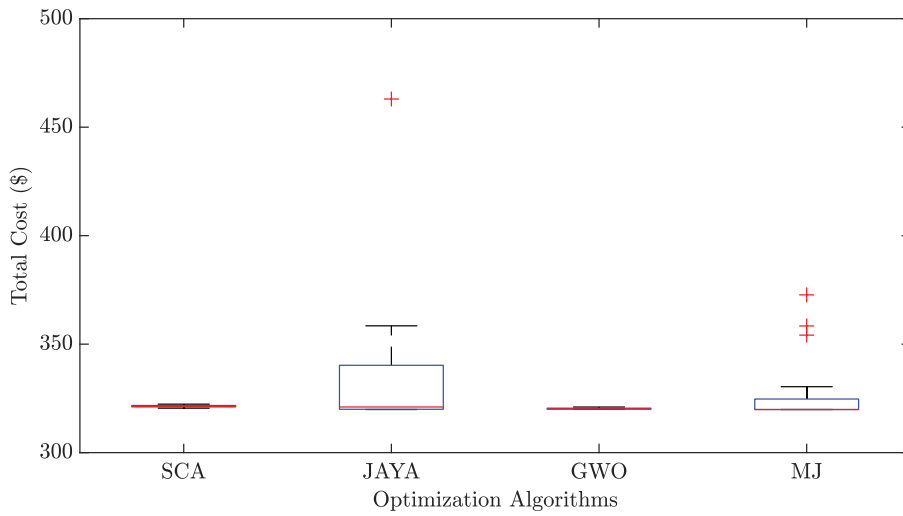


Figure 5. Box and whisker plot for the case R20P50.

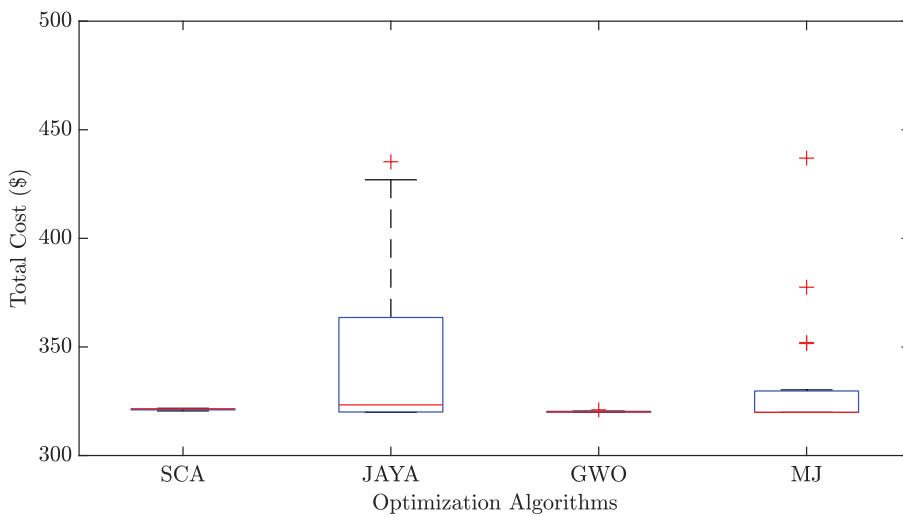


Figure 6. Box and whisker plot for the case R20P100.

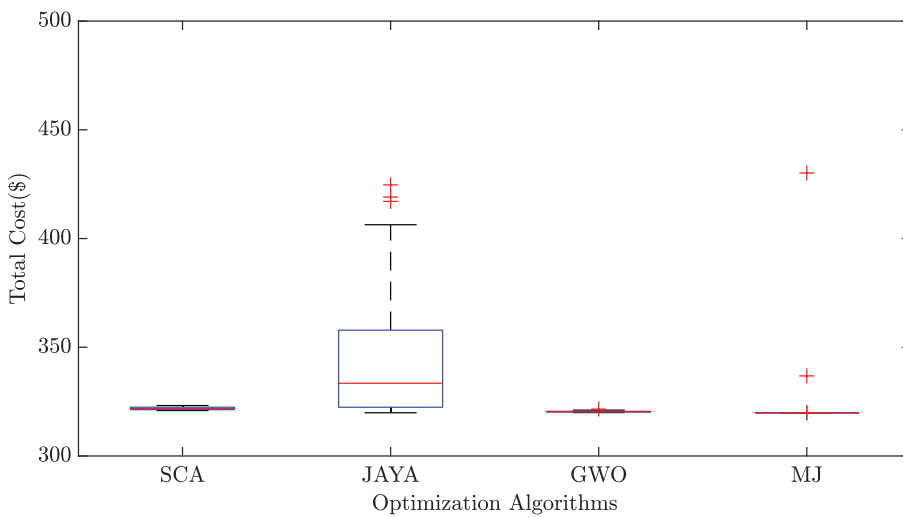


Figure 7. Box and whisker plot for the case R40P25.



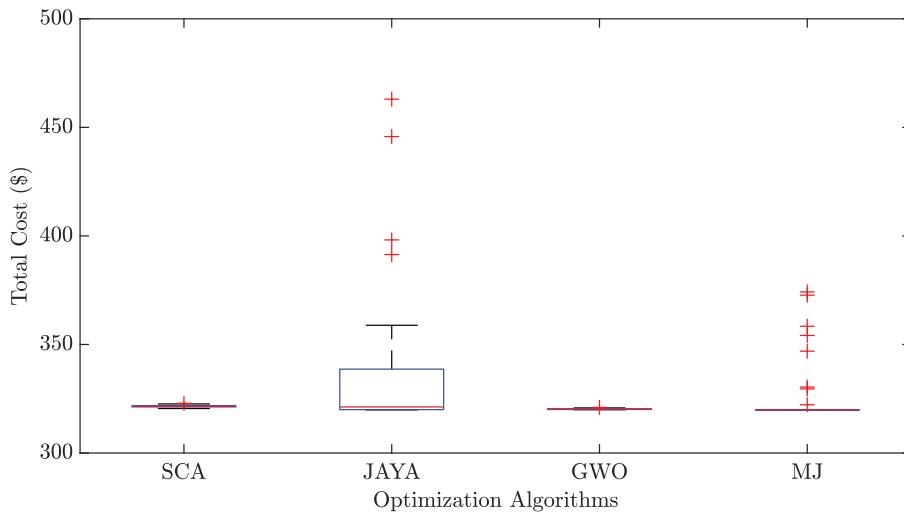


Figure 8. Box and whisker plot for the case R40P50.

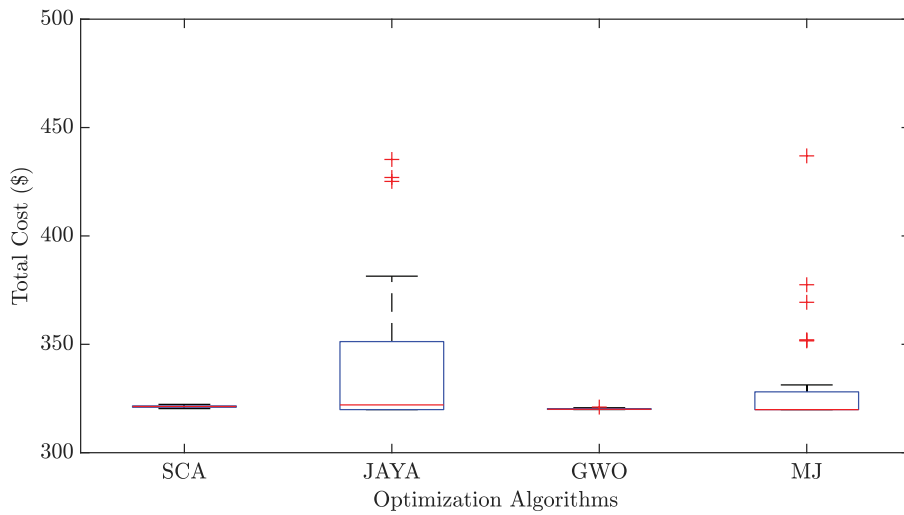


Figure 9. Box and whisker plot for the case R40P100.

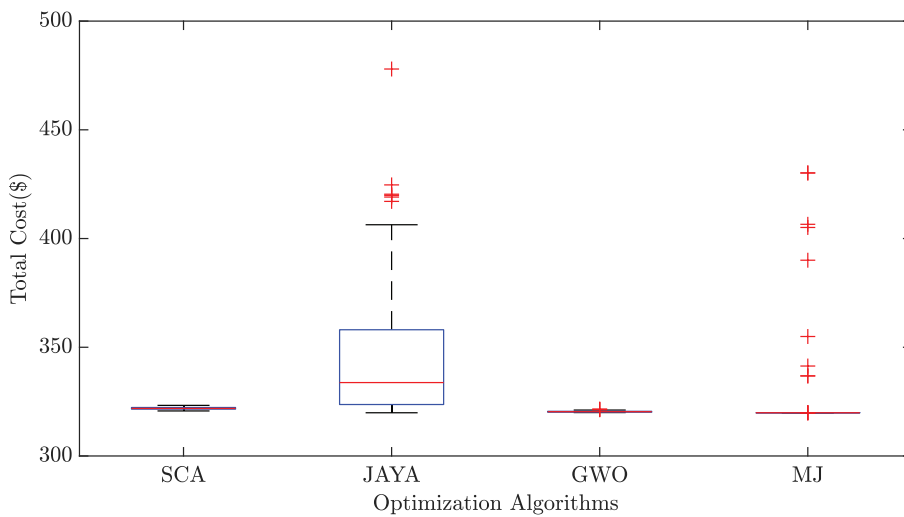


Figure 10. Box and whisker plot for the case R80P25.

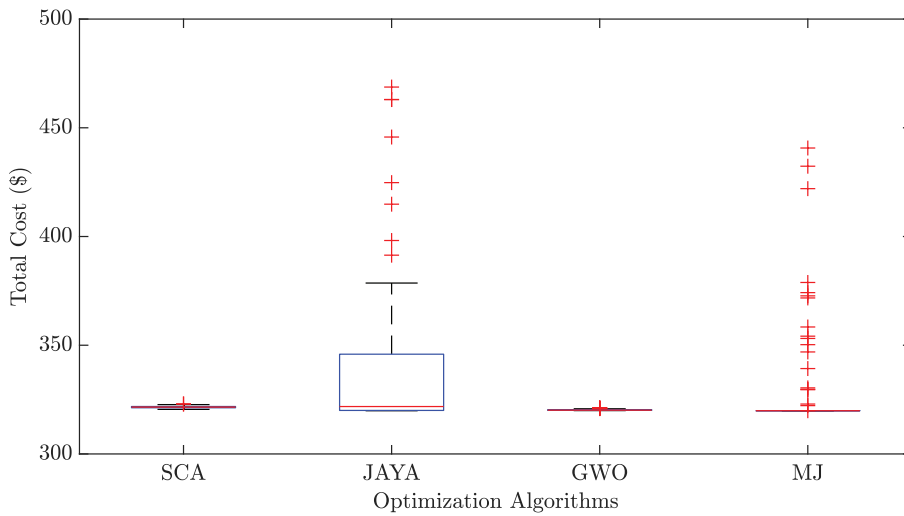


Figure 11. Box and whisker plot for the case R80P50.

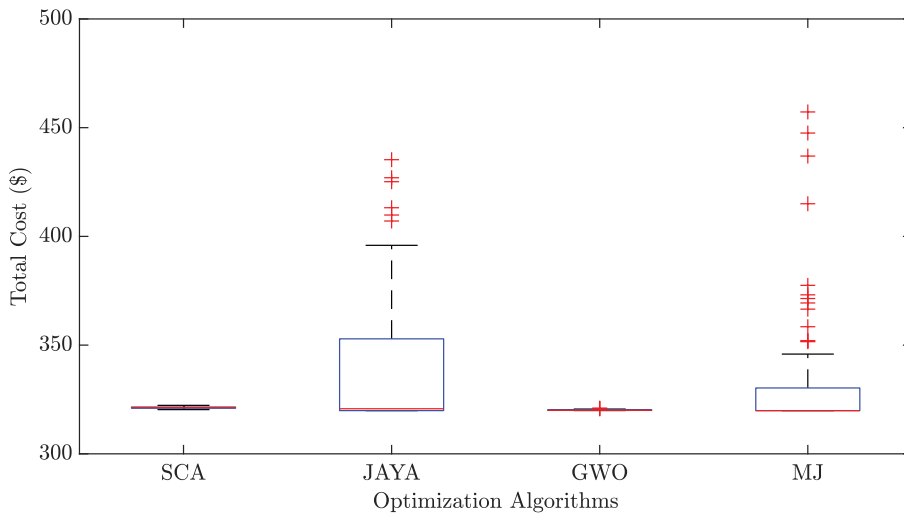


Figure 12. Box and whisker plot for the case R80P100.

A closer look at the box and whisker plots, Figure 4 to Figure 12, considering the comparing points mentioned above, can give a clear vision of the probability of MJ performance superior. Despite all this, so far, that was not enough to base a conclusive judgment on MJ’s superior performance, but it is just a piece of evidence that supports this direction.

Figure 13 to Figure 21 demonstrate the effectiveness of the different examined cases. Referring to these figures observed that all algorithms applied had a consistent behavior maximizing the effectiveness of the FTHE. All algorithms achieve an effectiveness of 0.78, which is the maximum effectiveness practically of this type of heat exchangers, as listed in [11]. SCA gave a slightly higher effectiveness value in some run points with differences not exceeding 0.009.

Table 5 shows the average execution time for each algorithm in different cases. The observation is that for all algorithms used, the execution time required increased with the increase in population sizes. MJ achieved almost the same execution time as Jaya, although all the above remarks led to decide that MJ tends to perform better. MJ, SCA, and Jaya gave close execution time values, while GWO execution time too higher compared to them.

Figure 22, Figure 23, and Figure 24 demonstrate the average rank for each algorithm through different cases. In Figure 22, for case R20P25, MJ obtained an average rank of 3.85 while SCA, JAYA, and GWO achieved 1.8, 1.35, and 3 respectively. Furthermore, for case R20P50, MJ achieved an average rank of 3.3 whereas SCA, JAYA, and GWO obtained 1.8, 2.05, and 2.85 respectively. For case R20P100, Figure 22, the average ranks are 2.88 for MJ, 1.95 for SCA,

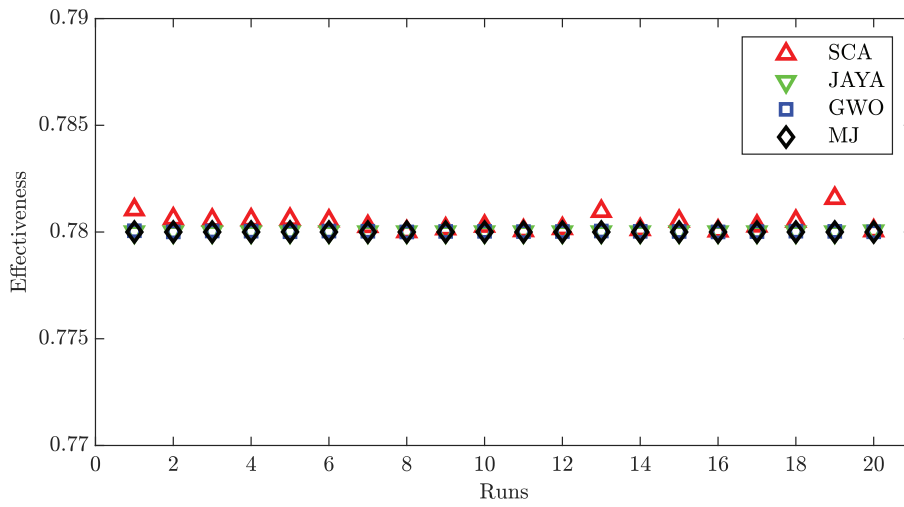


Figure 13. Effectiveness for case R20P25 through the 20 runs.

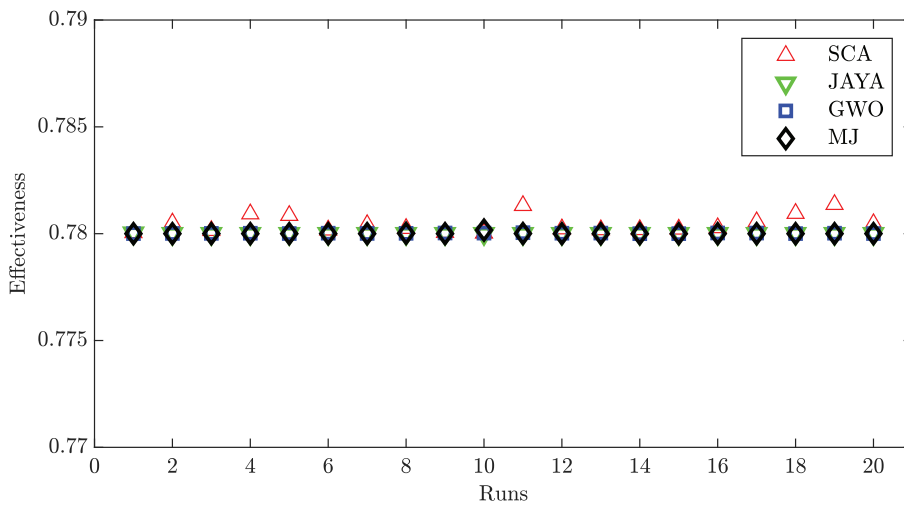


Figure 14. Effectiveness for case R20P50 through the 20 runs.

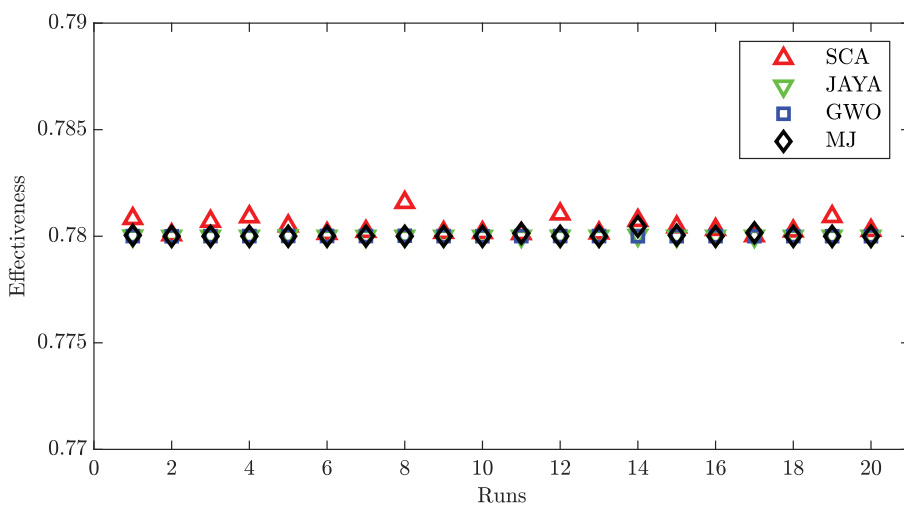


Figure 15. Effectiveness for case R20P100 through the 20 runs.

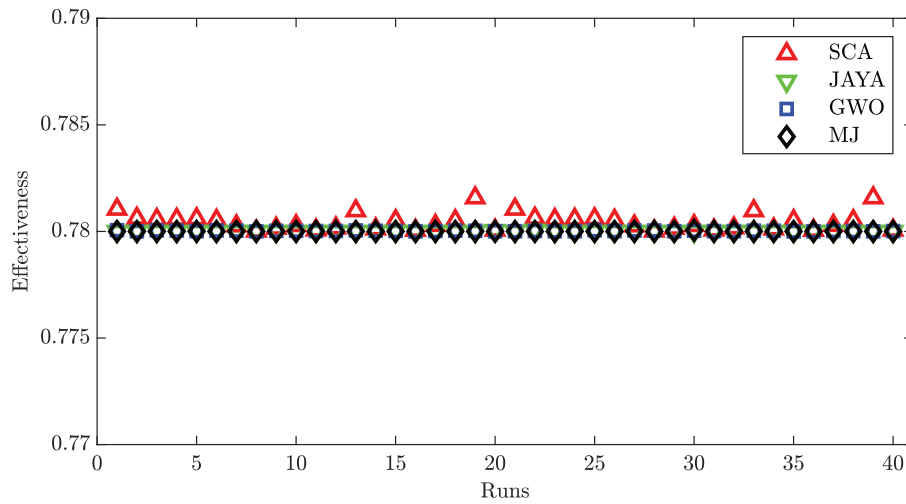


Figure 16. Effectiveness for case R40P25 through the 40 runs.

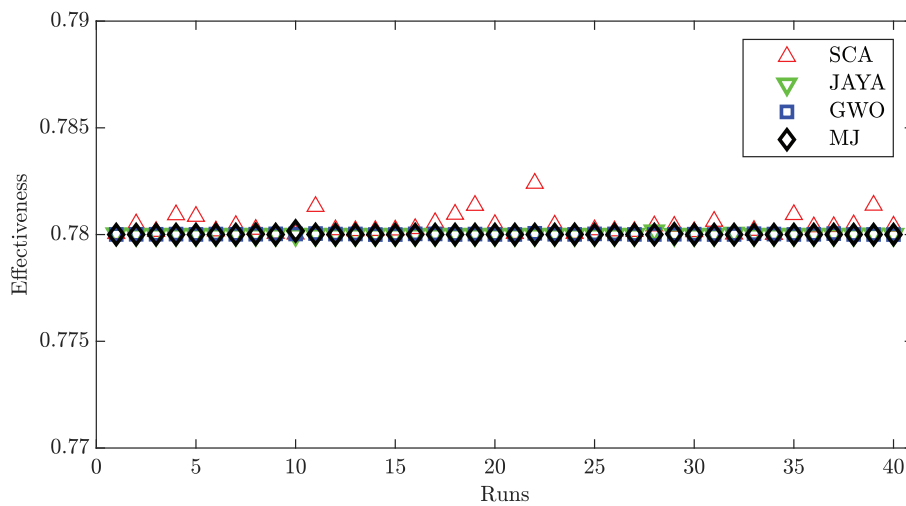


Figure 17. Effectiveness for case R40P50 through the 40 runs.

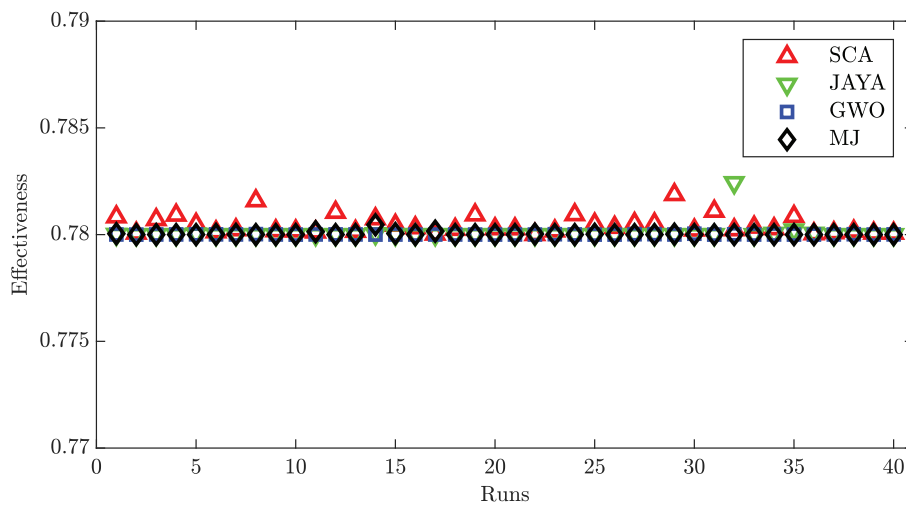


Figure 18. Effectiveness for case R40P100 through the 40 runs.

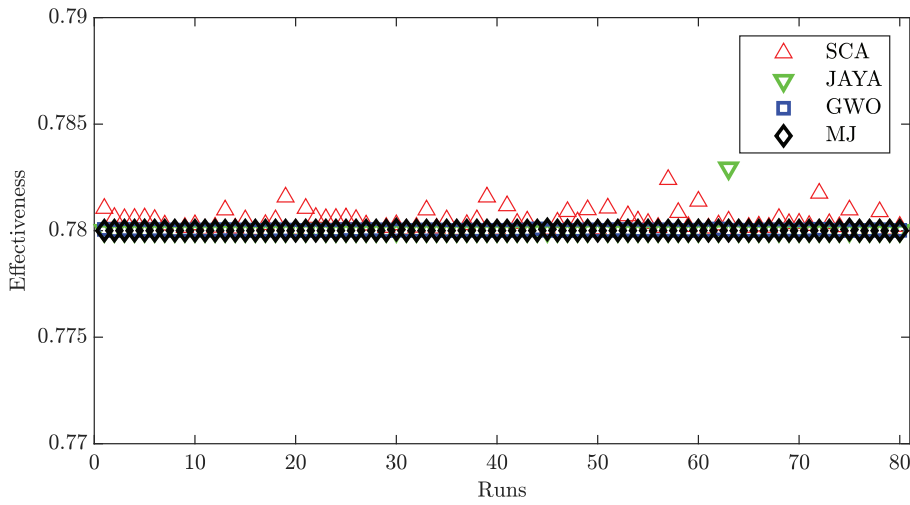


Figure 19. Effectiveness for case R80P25 through the 80 runs.

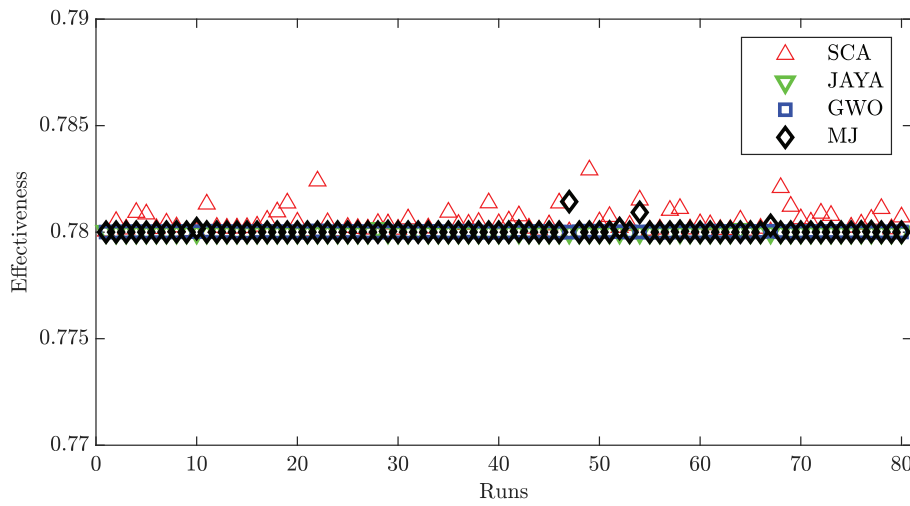


Figure 20. Effectiveness for case R80P50 through the 80 runs.

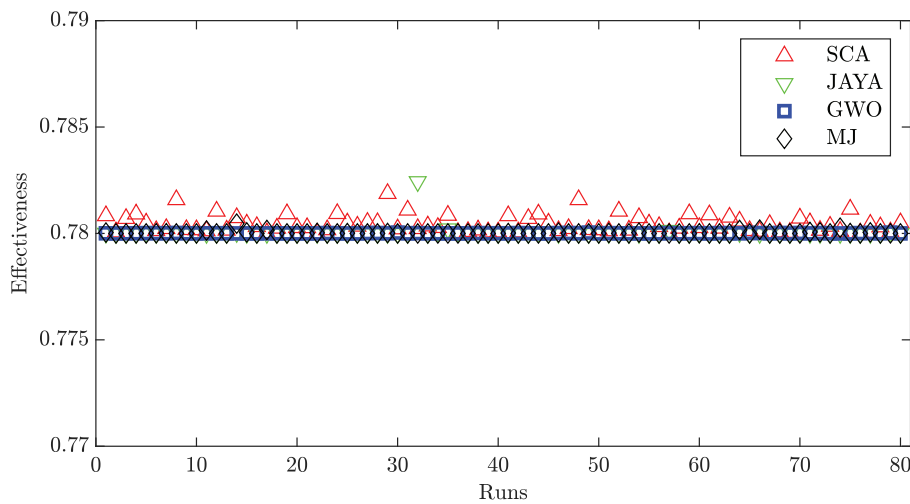


Figure 21. Effectiveness for case R80P100 through the 80 runs.

**Table 5.** Execution time for the different algorithms

Cases	Average Execution time (sec)			
	MJ	SCA	JAYA	GWO
R20P25	74.5754	75.1086	75.1179	113.9786
R20P50	77.3646	79.6686	78.1791	189.8736
R20P100	83.8043	85.0028	83.4192	457.6091
R40P25	74.3715	74.7718	75.3737	113.8110
R40P50	77.2536	80.0157	78.0086	191.9471
R40P100	83.6650	86.2867	84.7387	459.8875
R80P25	75.0291	76.1268	76.4525	115.0273
R80P50	77.3012	79.8940	78.0961	194.4597
R80P100	84.1018	85.9720	86.4110	463.0349

2.08 for JAYA, and 3.1 for GWO. The observation in Figure 22 is that MJ obtained an average rank higher than JAYA, SCA, and GWO through R20P25 and R20P50 cases. In case 3 (R20P100), GWO obtained an average rank higher than MJ, but still, the average rank of MJ is higher than JAYA and SCA.

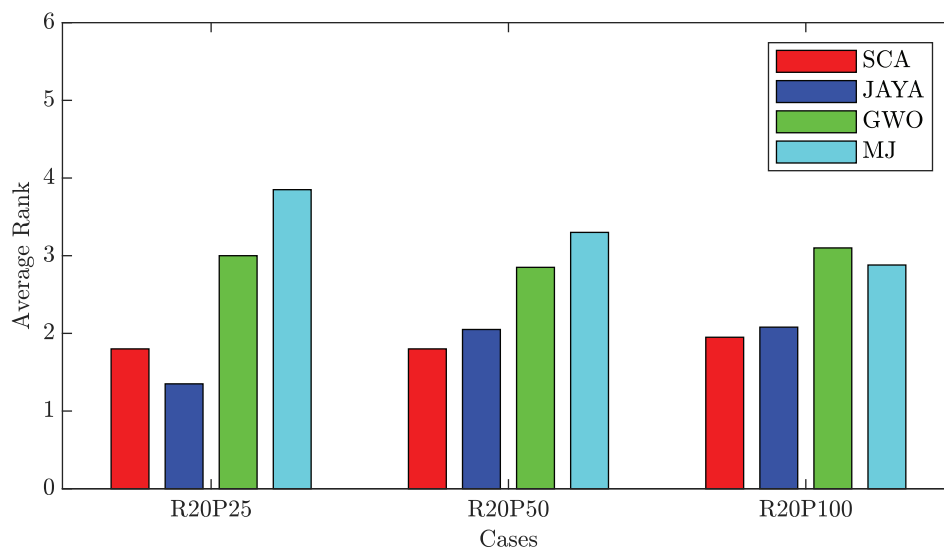
Figure 23 shows that in case 4 (R40P25) MJ obtained an average rank of 3.85 while SCA, JAYA, and GWO achieved 1.83, 1.35, and 2.98 respectively whereas in case 5 (R40P50) MJ achieved an average rank of 3.46 whereas SCA, JAYA, and GWO obtained 1.73, 1.99, and 2.83 respectively. In addition, in case 6 (R40P100), Figure 23, the average ranks are 3.14 for MJ, 1.83 for SCA, 2.14 for JAYA, and 2.90 for GWO. Figure 24 illustrates that in case R80P25 the average ranks are 3.7 for MJ, 1.93 for SCA, 1.34 for JAYA, and 3.04 for GWO whereas for case R80P50, MJ achieved an average rank of 3.31 while SCA, JAYA, and GWO obtained 1.83, 1.95, and 2.91 respectively. Furthermore, for case R80P100

Figure 24 the average ranks for MJ, SCA, JAYA, and GWO are 3.08, 1.8, 2.23, and 2.91. Figure 23 and Figure 24 show that MJ obtained higher average ranks during all examined cases than SCA, JAYA, and GWO. Again, this could indicate that MJ seems to perform better but still not enough to base a conclusive judgment.

Table 6 shows the ranks of the algorithms used to optimize the FTHE. From Table 6, the MJ algorithm achieved the highest ranks during all examined cases except in case 3 (R20P100), where GWO obtained a rank higher than the MJ rank with a difference of 7.26%. Based on that, MJ seems to obtain a better performance.

The final decisive comparing point that enables deciding regarding algorithms' superiority over each other is to perform a pairwise comparison (post hoc). The pairwise comparison was performed in this article using the least significant differences method (LSD). In other words, the absolute algorithms rank differences  $|R_i - R_j|$  must compare with the LSD values in each case. If  $|R_i - R_j|$  greater than LSD, one of the two algorithms compared outperforms another one. On the contrary, if  $|R_i - R_j|$  less than the LSD value indicates that the two compared algorithms perform equally.

Figure 25 to Figure 33 illustrate pairwise comparisons based on the LSD method for the different cases. Refer to Figure 25 (case R20P25) LSD value is 7.91. The observation is that MJ outperforms the JAYA, SCA, and GWO. As an arrangement, GWO ranked second in preference performance, while SCA and JAYA came in the third and fourth levels, respectively. In Figure 26 founded, MJ performs better than JAYA and SCA considering that LSD equals 14.11. MJ and GWO perform equally since their  $|R_i - R_j|$  value is less than LSD. In addition, the results in Figure 26 show that SCA and JAYA obtained the same performance in case R20P50. Besides this, results illustrate that GWO outperforms SCA and JAYA. A close look at Figure 27 demonstrates that in case



**Figure 22.** Algorithms average ranks through 20 runs.

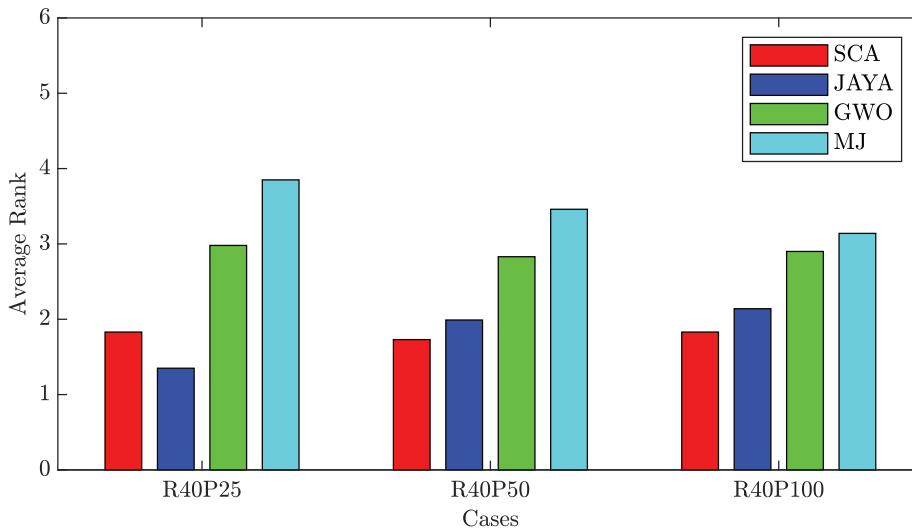


Figure 23. Algorithms average ranks through 40 runs.

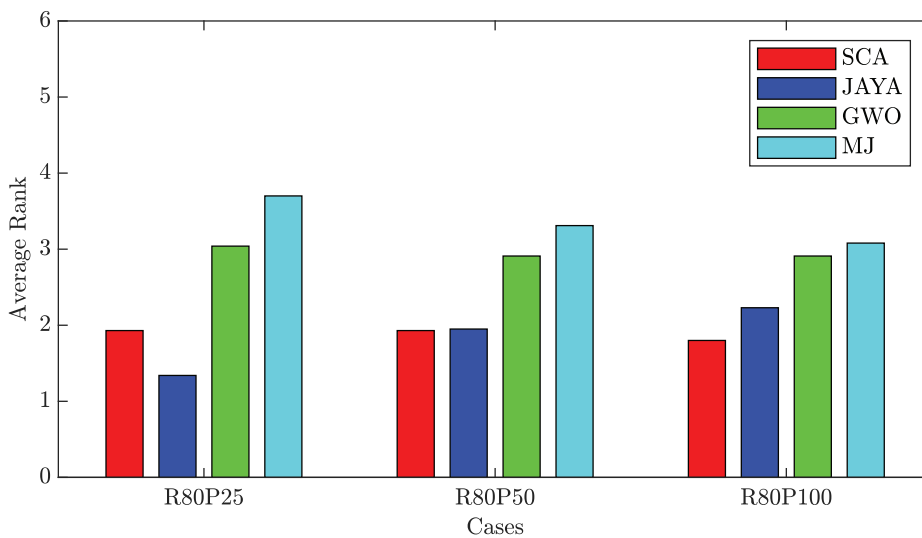


Figure 24. Algorithms average ranks through 80 runs.

Table 6. Algorithms ranks obtained in F-test

Cases	Algorithms Ranks			
	MJ	SCA	JAYA	GWO
R20P25	77	36	27	60
R20P50	66	36	41	57
R20P100	57.5	39	41.5	62
R40P25	154	73	54	119
R40P50	138.5	69	79.5	113
R40P100	125.5	73	85.5	116
R80P25	296	154	107	243
R80P50	265	146	156	233
R80P100	246	144	178	232

R20P100, MJ performs better than JAYA and SCA and performs equal to GWO performance. Also, this figure shows the equal performance of SCA and JAYA in addition to GWO being superior to SCA and JAYA.

The observations in Figure 28, considering the LSD value of 11.2, are that MJ is superior to all other algorithms used, followed in order by GWO, SCA, and JAYA, respectively. The domination of MJ performance over other algorithms is shown in Figure 29. Further remarks that can be recorded from this figure are the superiority of GWO over SCA and JAYA, besides the performance equality of SCA and JAYA. The MJ performs better than JAYA and SCA but is equal to the GWO performance in Figure 30. Additional recorded observations are JAYA and SCA equality in performance and the dominant performance of GWO over JAYA and SCA.

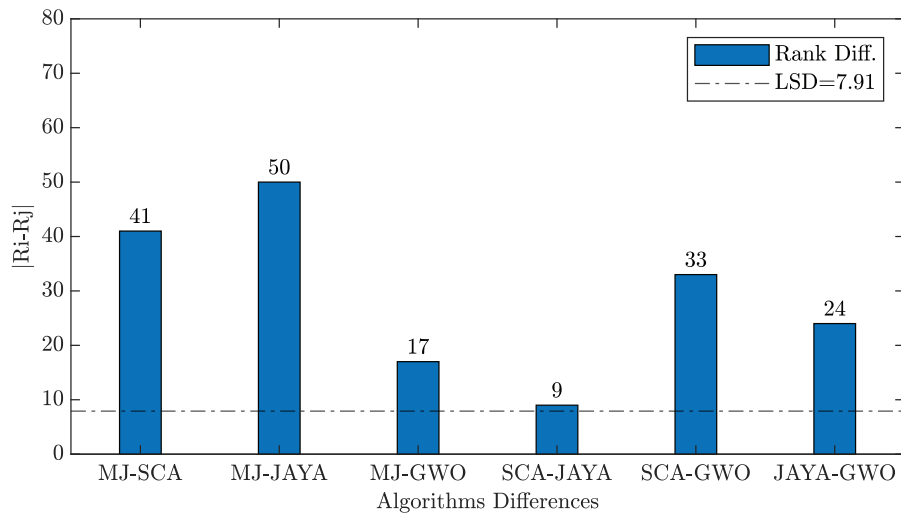


Figure 25. Pairwise comparison of case R20P25.

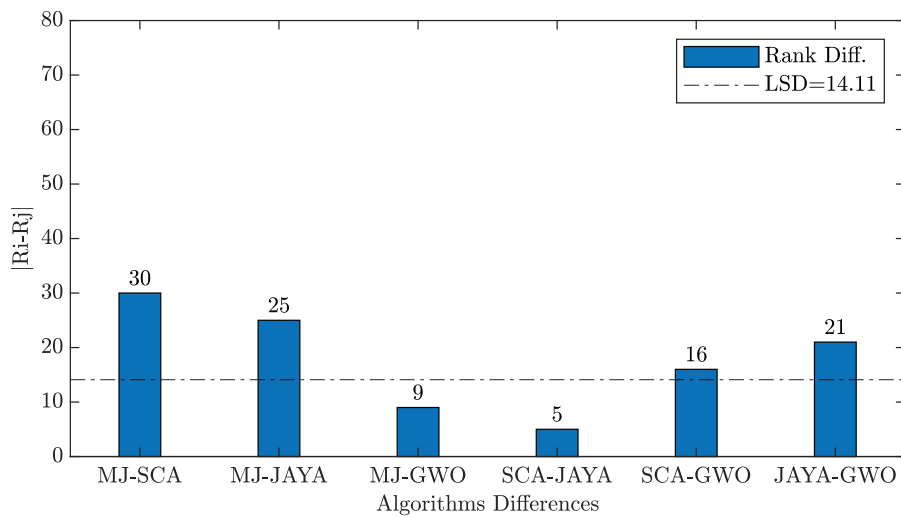


Figure 26. Pairwise comparison of case R20P50.

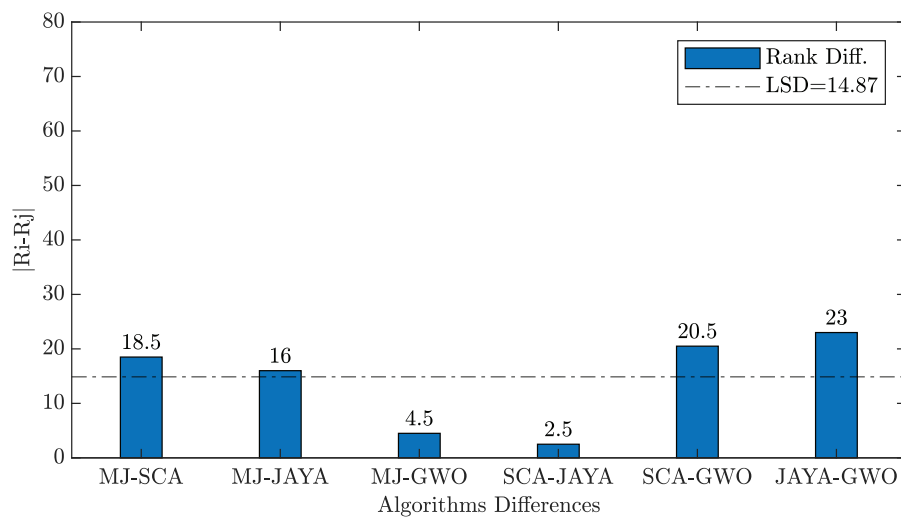


Figure 27. Pairwise comparison of case R20P100.



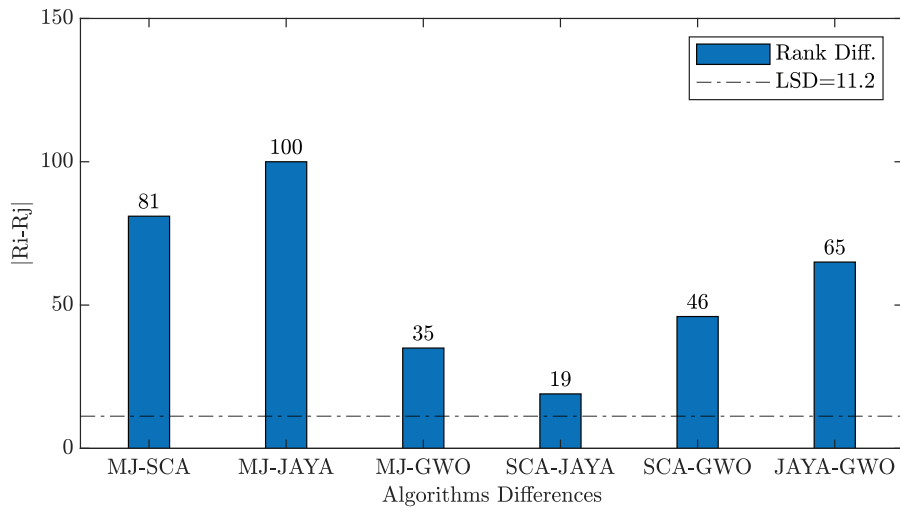


Figure 28. Pairwise comparison of case R40P25.

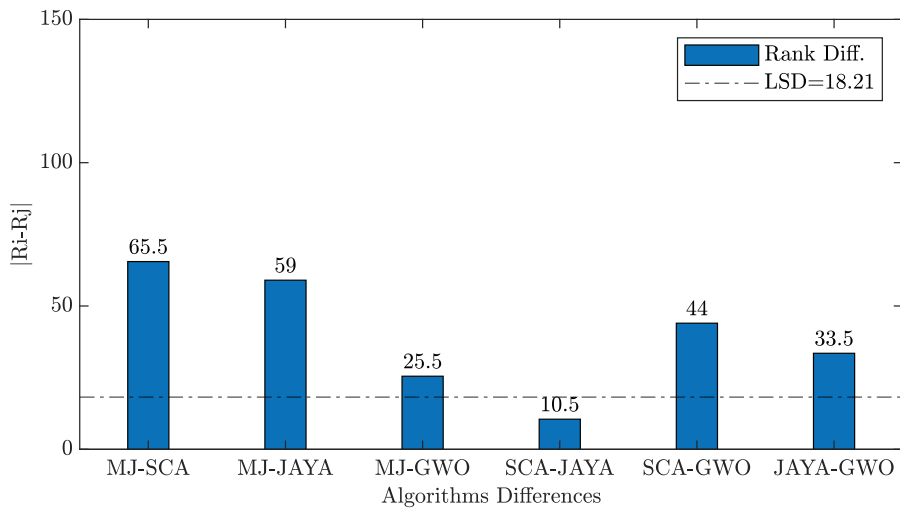


Figure 29. Pairwise comparison of case R40P50.

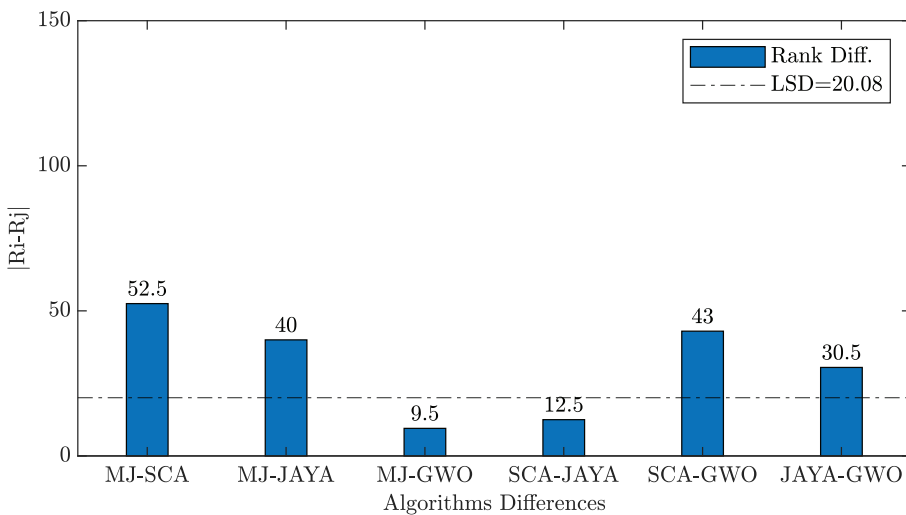


Figure 30. Pairwise comparison of case R40P100.

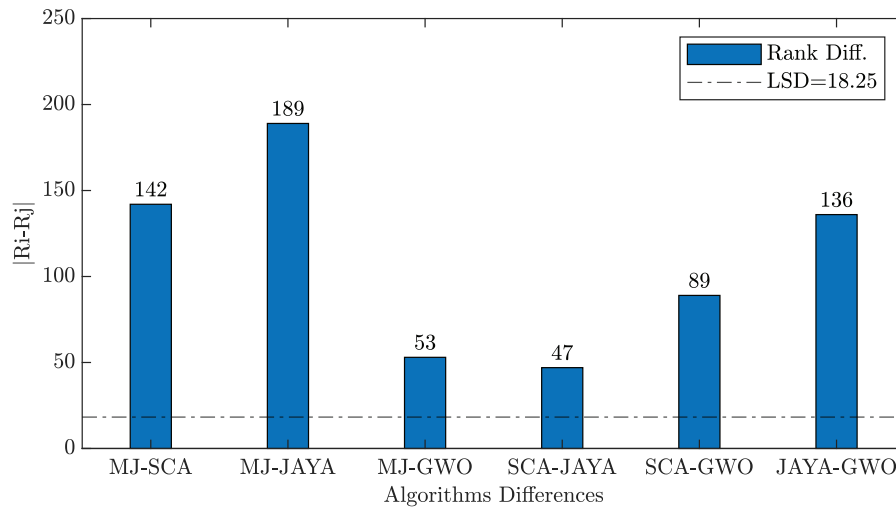


Figure 31. Pairwise comparison of case R80P25.

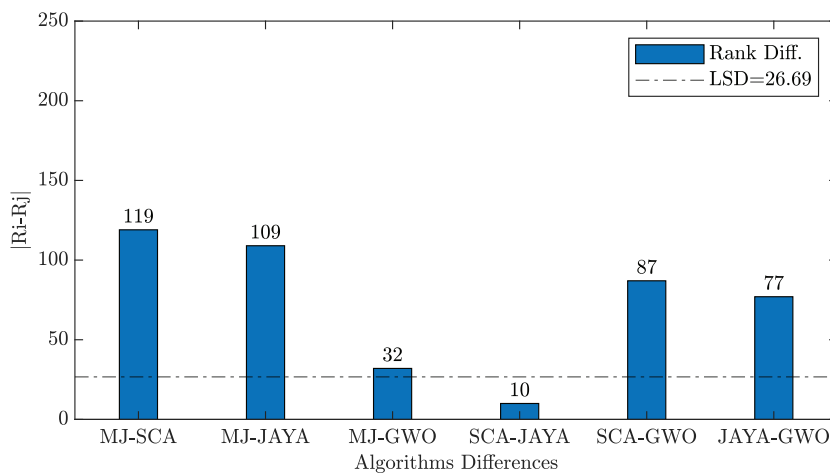


Figure 32. Pairwise comparison of case R80P50.

The supremacy of MJ compared to JAYA, SCA, and GWO was declared in Figure 31, followed in order by GWO edging on SCA and JAYA, respectively. The LSD of case R80P50, as shown in Figure 32, equals 26.69. Based on that, results indicate that MJ dominates SCA, JAYA, and GWO. On the other side, recorded remarks also show that GWO is supreme concerning JAYA and SCA, and SCA and JAYA equally perform. Finally, Figure 33 demonstrates that MJ outperforms JAYA and SCA and equals GWO in performance. Furthermore, GWO ranked advanced compared to JAYA and SCA, and JAYA supreme to SCA.

In the current work, optimization results achieved by all algorithms used (MJ, SCA, GWO, and JAYA) were validated using the results reported in [3] and [11]. Table 7 to Table 15 illustrate the comparisons of the Basic Geometrical Aspects in the current work results with the results available in the previous studies.

Furthermore, the validation process was extended by comparing the MJ results with samples of a larger dataset, including various benchmark functions and real-world optimization problems, because this will provide a more robust assessment of the algorithm’s performance and its generalizability across different scenarios. The benchmark functions selected are G06 and G09, whereas the real-world optimization problems selected are the pressure vessel design problem (PVD) and tension-compression spring design problem (TCS).

The G06 function is a minimization function with two variables ( $x_1$  and  $x_2$ ). The optimum solution of G06 function, at bounds  $13 \leq x_1 \leq 100$  and  $0 \leq x_2 \leq 100$ , is  $x^* = (14.095, 0.84296)$  where  $f(x^*) = -6961.81388$ . Details of the mathematical formulation of G06 are listed in [34].

The G09 function is a minimization benchmark function with seven variables ( $x_1, x_2, x_3, x_4, x_5, x_6$ , and  $x_7$ ).

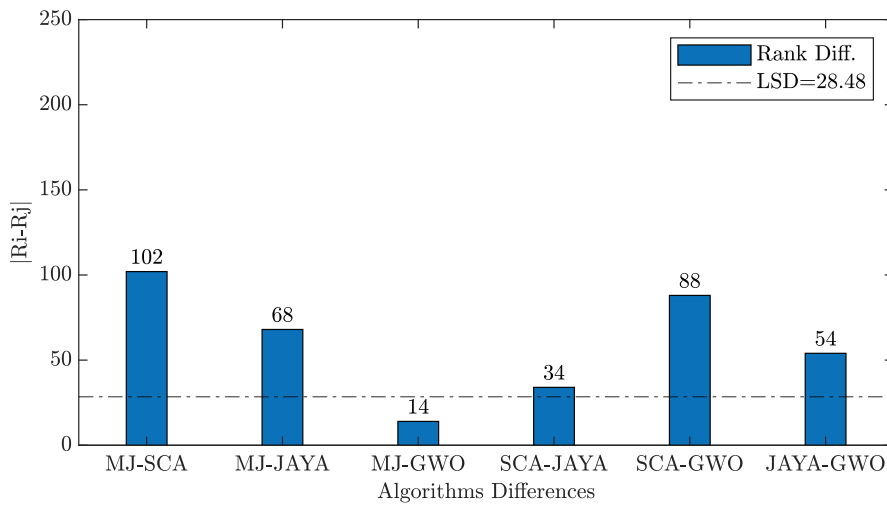


Figure 33. Pairwise comparison of case R80P100.

Table 7. Basic geometrical aspects comparison for case R20P25

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.1	10.0	10.2
$N_f$	315	-	315.2	311.8	317.1	317.3
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	553.3	560.5	558.8
$A_f/A_{tot}$	0.913	0.915	0.833	0.833	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.9	320.2	320.0
Maximum $\epsilon$	-	0.7795	0.7800	0.7802	0.7800	0.7800

Table 8. Basic geometrical aspects comparison for case R20P50

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.1	10.2	10.2
$N_f$	315	-	315.2	312.2	315.2	318.0
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	553.7	555.3	559.4
$A_f/A_{tot}$	0.913	0.915	0.833	0.833	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.5	319.9	319.9
Maximum $\epsilon$	-	0.7795	0.7800	0.7802	0.7800	0.7800

The optimum solution of the G09 benchmark function, at bounds  $-10 \leq x_i \leq 10$  where  $i=1, 2, \dots, 7$ , is  $x^* = (2.330499, 1.951372, -0.4775414, 4.365726, -0.6244870, 1.038131, 1.594227)$ , where  $f(x^*) = 680.6300573$ . More details about G09 mathematical formula are available in [34].

Table 16 shows the basic statistics of the MJ results in optimizing the G06 and G09 benchmark functions and its

comparison with other different algorithms, ABC [34] and DE [27], that optimized the same benchmark problems.

The PVD problem is a real-world classical optimization problem [35–38]. This problem has four variables  $(x_1, x_2, x_3, x_4) = (T_s, T_h, R, L)$ , and its objective function is to minimize the total cost of the pressure vessel. The mathematical formulation of the PVD problem is available in [35]. Table 17 shows the basic statistics of the MJ results in optimizing the

**Table 9.** Basic geometrical aspects comparison for case R20P100

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.3	10.2	10.2
$N_f$	315	-	315.2	313.8	315.2	313.1
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	553.0	555.3	552.2
$A_f/A_{tot}$	0.913	0.915	0.833	0.833	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.5	319.9	319.9
Maximum $\epsilon$	-	0.7795	0.7800	0.7803	0.7800	0.7800

**Table 10.** Basic geometrical aspects comparison for case R40P25

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.1	10.2	10.2
$N_f$	315	-	315.2	311.8	315.1	311.1
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$\alpha, m^2/m^3$	587	587.76	555.3	553.3	555.2	550.9
$A_f/A_{tot}$	0.913	0.915	0.833	0.833	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.9	319.8	320
Maximum $\epsilon$	-	0.7795	0.7800	0.7802	0.7800	0.7800

**Table 11.** Basic geometrical aspects comparison for case R40P50

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.1	10.2	10.2
$N_f$	315	-	315.2	312.2	315.2	318.0
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	553.7	555.3	559.4
$A_f/A_{tot}$	0.913	0.915	0.833	0.833	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.5	319.9	319.9
Maximum $\epsilon$	-	0.7795	0.7800	0.7800	0.7800	0.7800

**Table 12.** Basic geometrical aspects comparison for case R40P100

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.2	10.2	10.2
$N_f$	315	-	315.2	315.4	315.2	313.1
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	554.6	555.3	552.2
$A_f/A_{tot}$	0.913	0.915	0.833	0.834	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.3	319.9	319.9
Maximum $\epsilon$	-	0.7795	0.7800	0.7805	0.7800	0.7800

**Table 13.** Basic geometrical aspects comparison for case R80P25

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.2	10.2	10.2
$N_f$	315	-	315.2	325.3	315.1	318.9
$D_h, mm$	3.632	3.368	3.1	3.0	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	567.9	555.2	560.6
$A_f/A_{tot}$	0.913	0.915	0.833	0.834	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.7	319.9	319.9
Maximum $\epsilon$	-	0.7795	0.7800	0.7809	0.7800	0.7800

**Table 14.** Basic geometrical aspects comparison for case R80P50

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.1	10.2	10.2
$N_f$	315	-	315.2	312.2	315.2	318.0
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	553.7	555.3	559.4
$A_f/A_{tot}$	0.913	0.915	0.833	0.833	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.5	319.9	319.9
Maximum $\epsilon$	-	0.7795	0.7800	0.7802	0.7800	0.7800

**Table 15.** Basic geometrical aspects comparison for case R80P100

Geometrical Aspects	Ref [3]	GA Ref [11]	Current Work			
			MJ	SCA	JAYA	GWO
$D_o, mm$	10.2	-	10.2	10.2	10.2	10.2
$N_f$	315	-	315.2	315.4	315.2	313.1
$D_h, mm$	3.632	3.368	3.1	3.1	3.1	3.1
$t_f, mm$	0.33	-	0.33	0.33	0.33	0.33
$A, m^2/m^3$	587	587.76	555.3	554.6	555.3	552.2
$A_f/A_{tot}$	0.913	0.915	0.833	0.833	0.833	0.833
Minimum $C_{tot}, \$$	-	389.4	319.9	320.3	319.9	319.9
Maximum $\epsilon$	-	0.7795	0.7800	0.7805	0.7800	0.7800

PVD benchmark problem and its comparison with other different algorithms, GPEA [39], RUN [40], and EJAYA [35], that optimized the same benchmark problems.

Table 18 shows the optimum solution for the PVD problem obtained by MJ and its comparisons with the results data available in the previous works MSCA [37], ISOS [41], MBA [41], PSO [41], HFA [36], and EJAYA [35].

Regarding TCS, TCS is another practical real-world problem that is used as a benchmark in the field of validation of optimization algorithms [35-38]. This problem has three variables  $(x_1, x_2, x_3)=(d_w, d_c, P)$ , and the objective function of the TCS problem is weight minimization. More details about the mathematical formulations of the TCS problem are listed in [35].

**Table 16.** Statistical results obtained by MJ for G06 and G09 using 30 dependent runs through 16000 iterations

Benchmark functions	Statistics	Algorithms		
		MJ	ABC [34]	DE [27]
G06	Optimum G06	-6961.814	-6961.814	-6961.814
	Best	-6961.8139	-6961.814	-6954.434
	Worst	-6961.8139	-6961.805	-6961.814
	Mean	-6961.8139	-6961.813	-6961.814
	Median	-6961.8139	-	-
	Std	1.85 E - 12	0.002	-
G09	Optimum G09	680.63	680.63	680.63
	Best	680.9811	680.634	680.63
	Worst	686.8364	680.653	680.63
	Mean	682.4123	680.640	680.63
	Median	681.9277	-	-
	Std	1.48	0.004	-

**Table 17.** Statistical results obtained by MJ for PVD using 30 dependent runs through 16000 iterations

Algorithms	Best	Worst	Mean	Median	Std
MJ	5885.3328	5933.5313	5888.2240	5.8853328	9.5861
GPEA [39]	6059.708025	7445.205569	6277.343620	-	260.338256
RUN [40]	6059.716 662	7544.493 035	6871.604 953	-	605.2152
EJAYA [35]	5885.333	5894.777	5885.886	5885.366	1.734

**Table 18.** The optimum solution of the PVD problem obtained by MJ using 30 dependent runs through 16000 iterations

Algorithms	Optimum variables				Optimum objective function
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	
MJ	0.7781686	0.3846492	40.3196187	200.00	5885.3328
MSCA [37]	0.779256	0.399600	40.325450	199.9213	5935.7161
ISOS [41]	0.8125	0.4375	42.09844559585	176.63659584	6059.71433505
MBA [41]	0.7802	0.3856	40.4292	198.4964	5889.3216
PSO [41]	0.8125	0.4375	42.0984	176.6366	6059.7143
HFA [36]	8.17E-01	4.46E-01	4.23E+01	1.77E+02	6.15E+03
EJAYA [35]	0.778168665	0.38464918	40.319619559	199.99999545	5885.333

**Table 19.** Statistical results obtained by MJ for TCS using 30 dependent runs through 16000 iterations

Algorithms	Best	Worst	Mean	Median	Std
MJ	0.0126652	0.0128404	0.0126779	0.0126670	3.22 E - 5
GPEA [39]	0.012665	0.014071	0.013026	-	0.000344
HPSO [42]	0.012665	0.012719	0.012707	-	0.000015
EJAYA [35]	0.012665	0.012687	0.012668	0.012666	4.6331E-6

**Table 20.** The optimum solution of the TCS problem obtained by MJ using 30 dependent runs through 16000 iterations

Algorithms	Optimum variables			Optimum objective function
	$x_1$	$x_2$	$x_3$	
MJ	0.0516864	0.3566538	11.2927181	0.0126652
EO [36]	0.0516199100	0.355054381	11.38796759	0.012666
MSCA [37]	0.051668	0.356199	11.3207	0.0126670
OLCGOA [38]	0.051586809	0.354262809	11.4365114	0.012667456
ISOS [41]	0.051689061903120	0.356717759535058	11.288964594575669	0.012665
GPEA [39]	0.051860	0.360847	11.050894	0.012665
EJAYA [35]	0.05174315969	0.35802045837	11.2130152685	0.012665

Table 19 shows the basic statistics of the MJ results in optimizing the TCS benchmark problem and its comparison with other different algorithms, GPEA [39], HPSO [42], and EJAYA [35], that optimized the same benchmark problems.

Table 20 shows the optimum solution for the TCS problem obtained by MJ and its comparisons with the results data available in the previous works EO [36], MSCA [37], OLCGOA [38], ISOS [41], GPEA [39], and EJAYA [35].

## CONCLUSION

This article introduced a novel modified JAYA algorithm MJ aiming to improve performance. The modifications focused on controlling the generation of random numbers used in the position-updating equation and inserting an additional auxiliary weight to the equation, increasing the algorithm's capabilities to reach the best solution compared to the standard JAYA. The MJ algorithm optimized fins and tube heat exchangers, considering minimizing annual cost and maximizing effectiveness as an objective function. In addition to MJ, three other algorithms, namely the standard JAYA, SCA, and GWO, were used to compare the performance of the modified algorithm introduced. The comparing strategy started with one of the nonparametric tests (F-test) to decide whether there were significant differences between performances. Then, descriptive statistics measures were adapted to provide initial remarks about the leading algorithm. These were followed by the decisive step, which is pairwise comparisons. Based on the results listed and discussed previously could conclude that the novel modified JAYA algorithm MJ performs better than the standard JAYA, proving the validity of the earlier hypothesis that applying these modifications will improve performance compared to the standard JAYA. Furthermore, through all nine examined cases, MJ also shows supremacy over SCA. Regarding comparing MJ with GWO, MJ outperforms GWO in cases R20P25, R40P25, R40P50, R80P25, and R80P50, while performing equally with GWO in the rest of the cases with the advantage that MJ achieves less average execution time to find optimal solutions.

In future studies, authors suggest researchers apply the MJ algorithm to optimize various engineering optimization problems such as process optimization, electrical circuit design, structural optimization, thermal systems etc. Furthermore, the authors also recommend researchers seek to invent more novel optimization algorithms in addition to taking care of works that aim to hybridize the already existing algorithms to increase the capability to reach the optimum solution in less time.

## NOMENCLATURE

<i>MJ</i>	Modified JAYA
<i>SCA</i>	Sine Cosine Algorithms
<i>GWO</i>	Grey Wolf Optimizer
<i>FTHE</i>	Fins and tube heat exchangers
$L_L$	Hot flow path length
$L_W$	Cold flow path length
$L_H$	No-flow path length
$D_o$	Tubes outer diameter
$D_i$	Tubes inner diameter
$D_c$	Collar diameter
$P_t$	Transverse pitch.
$P_l$	Longitudinal pitch.
$t_f$	Fin thickness
$t_h$	Header thickness
$F_p$	Fin pitch
$F_s$	Fin spacing.
$N_f$	Number of fins
$X_i^t$	Immediate solution at the particular dimension ( $i$ ) and specific iteration ( $t$ )
$X_{worst}$	Worst solution
$X_{Best}$	Best candidate solution
$r_1, r_2$	Randomly generated numbers
$Z_1, Z_2$	Average absolute differences.
$D$	Number of design variables
$C_{tot}$	The total cost
$\varepsilon$	Heat exchanger effectiveness
$C_{in}$	Initial cost
$C_{op}$	Operating cost
$C_A$	Price of a unit of the surface area

$A$	Total surface area
$A_f$	Fins are
$k_{el}$	Price of the electricity
$\tau$	Operation hours
$\Delta P_a$	Pressure drop in the air side
$\Delta P_w$	Pressure drop in the water side
$V_t$	Volumetric flow rate
$\eta$	Pump/compressor efficiency
$C$	Heat capacity ratio
$NTU$	Number of transfer units
$U_o$	Overall heat transfer coefficient
$Re_w$	Reynolds number for water side
$Re_a$	Reynolds number for air side
$\epsilon_{max}$	Maximum practical effectiveness
$F$ -test	Friedman Test
$UQ$	Upper Quartile
$LQ$	Lower Quartile
$IQR$	Interquartile Range
$LSD$	Least Significant Differences
$T_s$	Thickness of the shell
$T_h$	Thickness of the head
$R$	Inner radius of the vessel
$L$	Length of the cylindrical section of the vessel
$d_w$	Wire diameter
$d_c$	Mean coil diameter
$P_c$	Number of active coils
$ABC$	Artificial Bee Colony
$DE$	Differential Evolutionary algorithm
$GPEA$	Grey Prediction Evolution algorithm
$RUN$	Runge Kutta optimization
$EJAYA$	Enhanced Jaya algorithm
$MSCA$	Multi-strategy enhanced sine cosine algorithm
$ISOS$	Improved Symbiotic Organisms Search algorithm
$MBA$	Mine Blast Algorithm.
$PSO$	Particle Swarm Optimization Algorithm
$HFA$	Human Felicity Algorithm
$HPSO$	Hybrid Particle Swarm Optimization Algorithm
$EO$	Equilibrium Optimizer.
$OLCGOA$	Orthogonal learning and chaotic exploitation based.

## ACKNOWLEDGMENT

This publication was supported by the Deanship of Scientific Research at Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia.

## AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

## DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

## CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## ETHICS

There are no ethical issues with the publication of this manuscript.

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