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Research Article

Modeling of parabolic collector (a new approach of concentration ratio calculation)

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ABSTRACT

Modern engineering challenges require the world to use renewable and environmentally friendly energy. One of the most important forms of renewable energy is solar energy. The parabolic collector is a popular collector used to absorb solar energy. In this study, a new approach is used to calculate the radiation concentration ratio in a parabolic collector. The concentration ratio is calculated from the ratio of the reflection beam to the incident beam radiation, and it depends on two main variables: the collector width (W) and the focal length (P). The model is tested and compared to results from previously published work. The comparison showed that the model results can be relied upon for accuracy and are compatible with published results. The results indicate that increasing the width of the collector (W) leads to an increase in the concentration ratio (RC), while the contrary is true when the focal length (P) increased. The collector efficiency minimum values were 19.3%, 21.07 %, 22.35% and 23.33% at concentration ratios of 69, 80, 103 and 148 in line with the focus length values of 0.6m, 0.7m, 0.8m and 0.9m, respectively. The developed model is applied according to the conditions of Basra, Iraq (47.78° longitude and 30.5° latitude).

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INTRODUCTION

Clean energy has played an important role in energy generation several decades ago, resulted in reduce the carbon emissions and create new investment opportunities. One of the major types of the clean energy is the solar energy, which can be generated either from sun light or from sun radiation. There are several technologies that can take advantages of solar radiation such as parabolic solar collectors. Collectors have taken on a large number of concerns from researchers around the world. Milad Tajik Jamal-Abad et al [1] experimentally studied the process of determining the efficiency of a solar parabolic

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trough collector. The metal foam is used in the absorber to improve the heat transfer and increase the efficiency of the parabolic trough collector (PTC). Dongming Zhao et al [2] developed a model of performance simulation of a PTC. This model included on the optics cone with non-parallel rays, geometric concentration ratios (GCs), the glass tube transmissivity, the absorber tube absorptance and the collector surface reflectivity. J. Subramani et al [3] used the nano-fluid technology to improve the efficiency of the parabolic trough solar collector. Aggrey Mwesigye et al [4] proposed a numerical study to find the influence of high thermal conductivity of the work fluid, on the performance indicators of the parabolic trough solar collector. Ali Khosravi et al [5] used computational fluid dynamics to evaluate and study the performance of a PTC. The work included the enhancement of the convective heat transfer of Fe_3O_4-Therminol 66 ferrofluid under magnetic field (0–500 G). Caiyan Qin et al [6] presented a new approach for a transparent direct-absorption parabolic-trough solar collector (DAPTSC) by applying a reflective coating on the upper half of the inner glass tube outer surface such that the optical path length is doubled compared to that of the transparent DAPTSC. Xueling Li et al [7] presented a two-dimensional numerical model to analyze the performance of PTC. This model included the effects of all the parameters that influence the performance of the collector. The parameters investigated in their study are inclination angle, collecting temperature, surface emissivity and aperture width on the heat loss. Wanjun Qu et al [8] conducted an experimental study of controlling and minimizing the cosine loss by controlling horizontal rotation of the PTC. They found out that the surface azimuth is influenced by the horizontal position. Therefore, reducing the cosine loss resulted in reducing the solar incidence angle. S. Yang et al [9] introduced a three-dimensional (3D) model that calculates the performance of a parabolic trough solar collector. The model is based on thermodynamic and heat transfer equations in addition to empirical correlations. All of which were linked and simplified to produce ordinary differential equations represent this model.

From the review of previous research, it is shown that the subject of the solar parabolic collector is of great importance in the scientific community. Therefore, in this research, a mathematical model is introduced to simulate the work of the solar parabolic collector in the conditions of Basra - Iraq. In this model, attention is paid to the finding of the performance of this collector, particularly, finding its efficacy. As well as propose a new method for calculating the concentration ratio for the collector.

The paper is structured as follows, mathematical model and thermal model are presented in section 2 and section 3 respectively. The results are analyzed in section 4 followed by concluding the work in section 5.

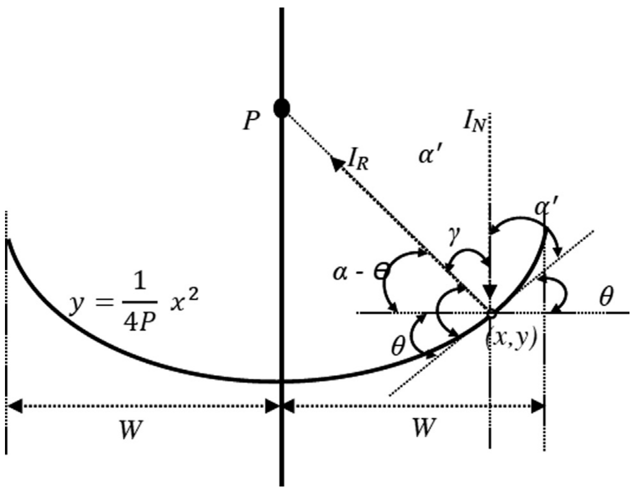

Figure 1. sketch of the parabolic collector.

MATHEMATICAL MODEL

PTC is made of reflecting material, that shaped as a parabolic collector to absorb the sun energy and reflect it to the focal line along the collector as shown in Figure 1 below. In this work the PTC is considered as a parabola with the following assumptions:

1) The angle of incidence is equal to the angle of reflection.
2) The surface of the collector has absorption coefficent (β).
3) The collector has a perfect parabolic surface acoording to Equation $y = \frac{1}{4P}x^2$.

The concentration ratio (R_c) of the solar collector is an important ratio or variable in the functioning and efficiency of PTC. Concentration ratio (Rc) can be defined as the ratio of the summation reflection beam to the incident radiation beam on the collector surface.

The angle (α) can be calculated as:

$$\left.\begin{array}{l} \alpha = \tan^{-1}\left(\dfrac{w}{P - \dfrac{1}{4P}w^2}\right) \; if\ \alpha < 90^\circ \\[2ex] \alpha = \tan^{-1}\left(\dfrac{w}{\dfrac{1}{4P}w^2 - P}\right) \; if\ \alpha < 90^\circ \end{array}\right] \quad (1)$$

From the basic mathematical definitions of the first derivative, the angle of the slope of the tangent curve of the pieces can be calculated as (see Figure 1):

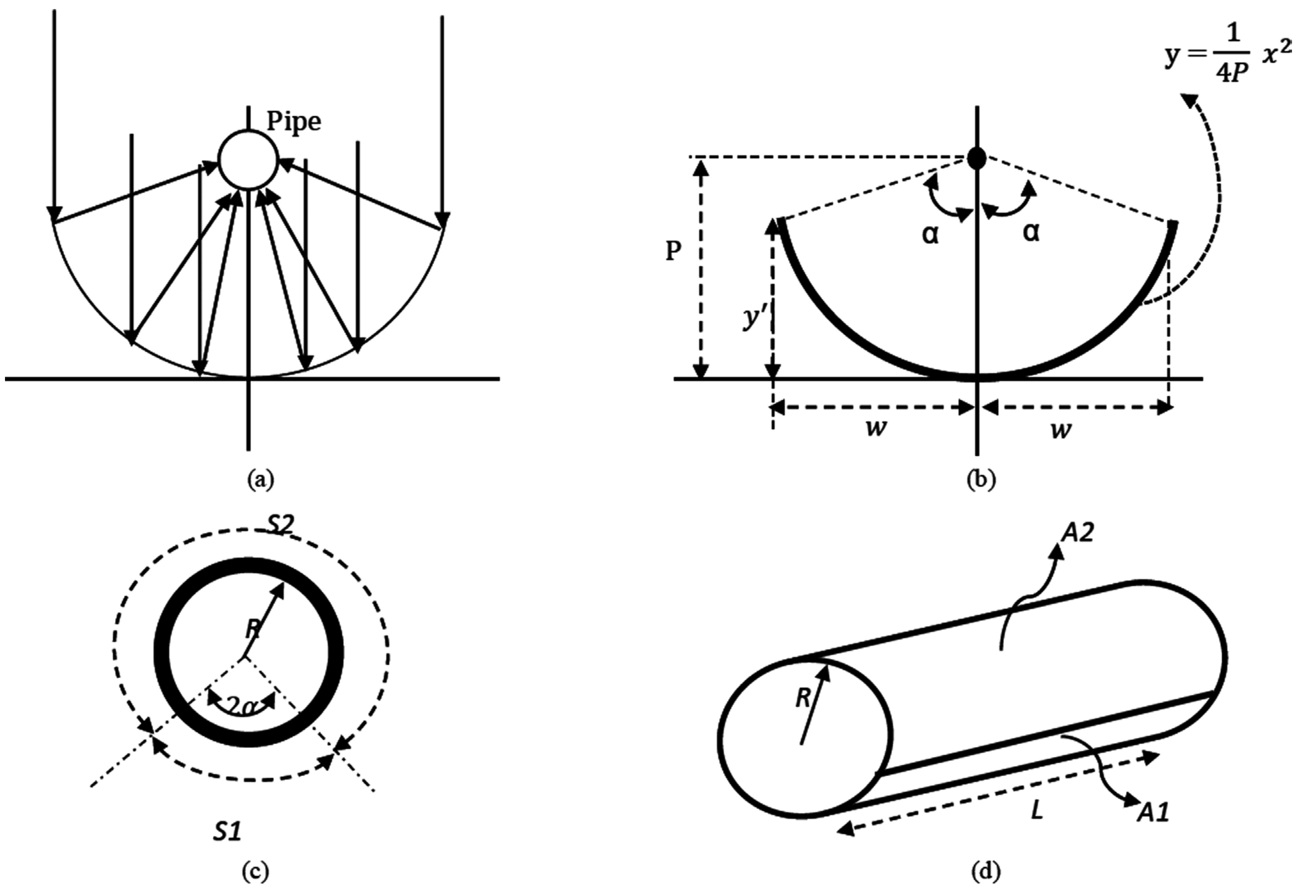

Figure 2. The details of parabolic collector (a) principle parabolic collector (b) geometry of collector (c) side view of the pipe (focus) (d) the upper and lower surface area of pipe.

$$\left.\begin{aligned} \left[\tan(\theta) = \frac{dy}{dx}\right]_{at\ point(x,y)} &= \frac{1}{2P}x \\ \therefore \theta = \tan^{-1}&\left(\frac{1}{2P}x\right) \end{aligned}\right\} \tag{2}$$

Referring to Figure 1 with the first hypothesis, the following relationship can be written:

$$\gamma = 180 - 2\alpha = 180 - 2(90-\theta) = 2\theta \tag{3}$$

By substituting Equation 4 in Equation 5, it produces:

$$\gamma = 2\tan^{-1}\left(\frac{1}{2P}x\right) \tag{4}$$

In vectors form (Figure 1):

$$dI_R = I_N \cos(\gamma) = I_N \cos\left(2\tan^{-1}\left(\frac{1}{2P}x\right)\right) \tag{5}$$

Equation 5 represent the reflected beam at point (x,y), when we need summation all beams reflected from the parabolic, then integration of Equation (5) in the range (–W to W) resulted in:

$$I_R = \int_{-W}^{W} I_N \cos\left(2\tan^{-1}\left(\frac{1}{2P}x\right)\right)dx \tag{6}$$

Integrated the Equation 6 can be explained below (Equations 7,8, and 9)

$$\begin{aligned} I_R = I_N &\left[\frac{8\tan^{-1}\left(\frac{px}{4}\right)}{P} - x\right]_{-W}^{W} = \\ I_N &\left\{\frac{8\left(\tan^{-1}\left(\frac{pw}{4}\right) - \tan^{-1}\left(\frac{-pw}{4}\right)\right)}{P} - (W-W)\right\} \end{aligned} \tag{7}$$

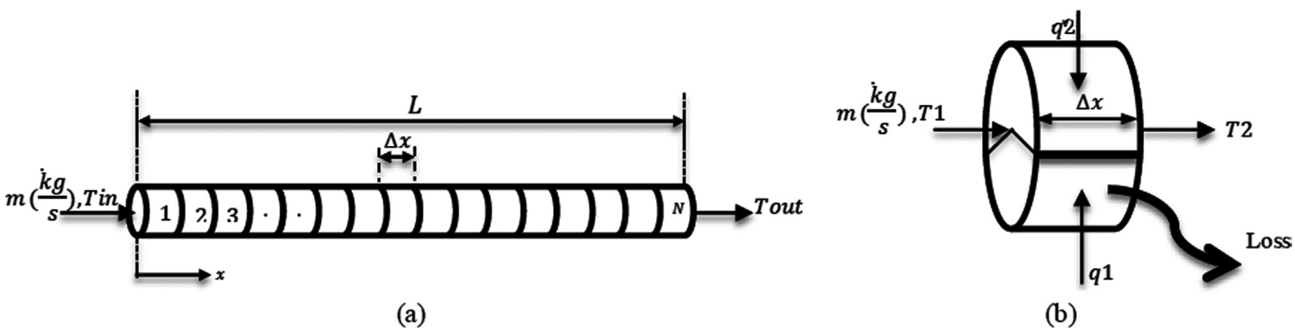

Figure 3. (a) collector pipe (b) the element.

$$I_R = I_N \left\{ \frac{8}{P} \left(\tan^{-1}\left(\frac{PW}{4} \right) - \tan^{-1}\left(\frac{-PW}{4} \right) \right) \right\} = I_N \left(\frac{16}{P} \right) \tan^{-1}\left(\frac{PW}{4} \right) \quad (8)$$

Then, the concentration ratio is:

$$Rc = \frac{I_R}{I_N} = \left(\frac{16}{P} \right) \tan^{-1}\left(\frac{PW}{4} \right) \quad (9)$$

THERMAL MODEL

The one-dimensional (1D) heat transfer model is adapted in this study. The temperature gradient is considered to be significant only in the axial direction of the HCE rather than in radial and circumferential directions. In fact, since the tube is insulated (that is, heat losses in the radius are neglected) one can only consider the temperature gradient to be in the axial direction. Therefore, it is possible to take an element of the tube and perform a thermal balance on it as in Figure 2.

Energy Balance:

The parabolic collector is used to heat water (Figure 3), the energy supplied to the water can be evaluated as:

$$\dot{m}\, cp\ (T2 - T1) + Loss = q1 + q2 \quad (10)$$

$$\left. \begin{aligned} q2 &= I_N \Delta A2\ \beta \\ q1 &= I_c \Delta A1\ \beta \end{aligned} \right] \quad (11)$$

Where:

I_N the solar radiation intensity (W/m^2).

I_c the concentrated intensity radiation (W/m^2).

β absorption coefficient.

h heat transfer coefficient (W/m^{2}c^o).

$\bar{T}, T\infty$ average and ambient temperature (c).

The heat losses in Equation 10 included two types of losses, convection and radiation loss as:

$$q_{con,loss} = h_o A_o (\bar{T} - T_\infty) \quad (12)$$

$$q_{rad,loss} = \varepsilon\sigma(\bar{T}^4 - T_\infty^4) \quad (13)$$

Where h_o external convective heat transfer coefficient. $\varepsilon\sigma$ are emissivity of the tube and the Stefan-Boltzmann constant being 5.67×10^{-8} W/m^2 K^4 respectively.

If the angle (α) in radian, then the length of both sectors s_1 and s_2 evaluated as:

$$\left. \begin{aligned} s1 &= 2\alpha R \\ s2 &= (2\pi - 2\alpha)R \end{aligned} \right] \quad (14)$$

The upper and lower areas of the element are:

$$\left. \begin{aligned} \Delta A2 &= (1-\alpha)2\pi R\Delta x \\ \Delta A1 &= 2\pi R\Delta x \end{aligned} \right] \quad (15)$$

Where: $\Delta A1$ is the surface area of the element on which the reflected rays are concentrated from the collector, and $\Delta A2$ is the surface area of the element which exposed to direct solar radiation.

The element temperature is:

$$\bar{T} = \frac{T1 + T2}{2} \quad (16)$$

Substituting Equations 11, 12, 13, 14, 15 and 16 in Equation 10 we get:

$$\dot{m}\, cp(T2 - T1) + h_o A_o \left(\frac{T1+T2}{2} - T_\infty \right) + \varepsilon\sigma \left(\left(\frac{T1+T2}{2} \right)^4 - T_\infty^4 \right) = I_c\ (2\pi R\Delta x)\beta + I_N\ ((1 - \alpha)2\pi R\Delta x)\beta \quad (16)$$

solving above for unknown T2.

The external heat transfer coefficient is [10]:

$$h_o = \frac{Nuk}{D} \quad (17)$$

For heat convection with no wind, the expression given by Churchill and Chi [11] is recommended for horizontal cylinder under natural convection [10].

$$Nu = \left[0.6 + 0.387 \left\{ \frac{Ra_D}{\left[1 + \left(\frac{0.559}{\Pr} \right)^{\frac{9}{16}} \right]^{\frac{16}{9}}} \right\}^{\frac{1}{16}} \right]^{\frac{1}{2}} \tag{18}$$

For heat convection with wind, the average Nusselt number recommended for a cylinder in cross flow is given by [11]:

$$Nu = c\, Re_D^m \Pr^n (\Pr/\Pr_w)^p \tag{19}$$

The constants, m and n suggested for this equation are given by [11]. The value of p depends on the heat flux direction: $p = 0.25$ for fluid heating and $p = 0.2$ for fluid cooling.

The efficiency of the collector is:

$$\eta = \frac{output\ energy}{input\ energy} = \frac{\dot{m}cp(Tout - Tin)}{I_N \times L * 2W} \tag{20}$$

RESULTS AND DISCUSSION

Validation

The developed model in the previous section is applied on the data and variables shown in Table 1 below and compared to the available results in [12]. This model determines the solar radiation, heat gain, thermal efficiency, and outlet temperature with a 30-minute interval.

Figure 4 shows a comparison of the outlet temperature of the collector from this study with the researcher's results [12]. From the Figure, it can be seen that there is a good correlation between the proposed model and the results from [12], and the biggest error is 2.74%.

Figure 5 denoted the comparison between the thermal efficiency evaluated by present work with that founded by Mohanad [12]. A good agreement of the results is predicted in this Figure 5 with maximum error of 5.6%.

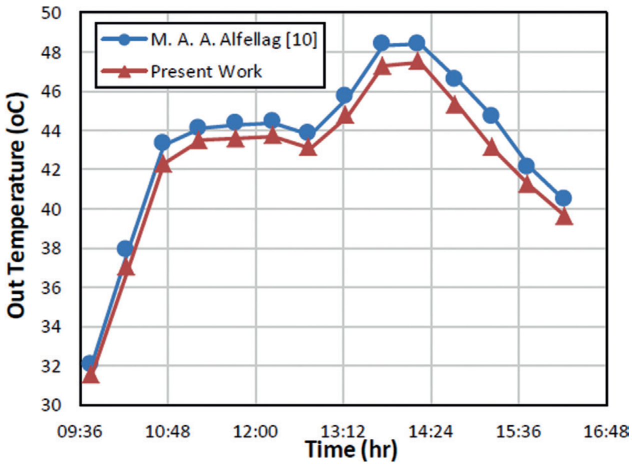

Figure 4. Variation of outlet temperature with time.

Table 1. The data used to comparison [12]

Time	Tin (°C)	Tout (°C)	Volumetric Flow rate (GPM)	Ambient temperature (°C)	wind speed (m/s)
09:45	30.4	32	1	20	2.2
10:15	35.3	37.9	1	21	2.2
10:45	40.9	43.3	1	22.2	2.8
11:15	41.5	44.1	1	23.3	2.8
11:45	41.6	44.3	1	23.8	2.8
12:15	41.3	44.4	1	25	3.1
12:45	40.3	43.3	1	26	3.1
13:15	42.5	45.7	1	26	3.1
13:45	45.3	48.35	1	26.6	3.8
14:15	45.4	48.4	1	26.6	3.8
14:45	43.9	46.6	1	26.6	3.1
15:15	42.3	44.7	1	26.6	3.1
15:45	40	42.1	1	26	3.7
16:15	38.7	40.45	1	25.5	3.7

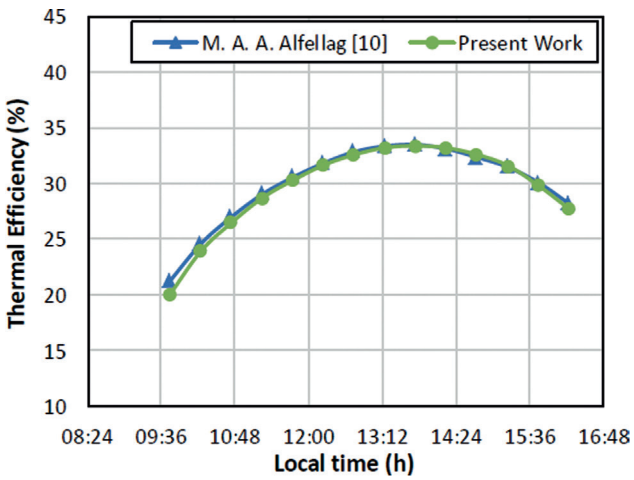

Figure 5. Variation of thermal efficiency with time.

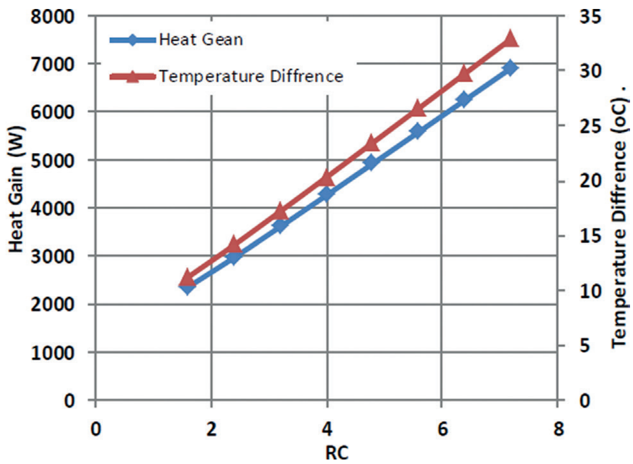

Figure 7: Effect the concentration ratio (RC) on both heat gain and temperature difference.

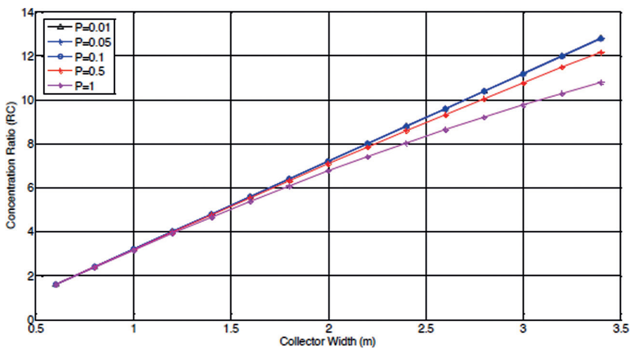

Figure 6. Variation the concentration ratio (RC) with both collector width (w)and focus length (P).

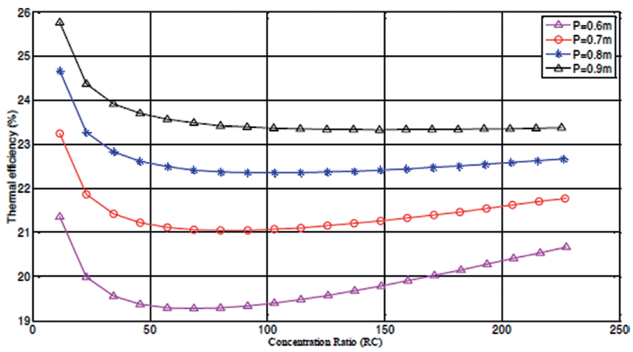

Figure 8. The thermal efficiency effected by concentration ratio.

Concentration Ratio

Figure 6 shows the concentration ratio calculated from Equation 9, where it is dependent on collector width (W) and the focus length(P). From the Figure, it can be seen that collector width has more effect than focus length on concentration ratio. The collector width correlated positively with the concentration as the concentration ratio increase when the collector with increase. While the opposite is true of the focus length. This means when the value of focus length (P) is fixed, and when the value of the collector width (W) is increased, the concentration ratio will increase too. This is a logical thing, as when increasing the width of the collector, the surface area of the collector will increase. Then the amount of radiation incident on the surface of the collector will increase, which leads to an increase in the reflected radiation, thus increasing the concentration ratio. It should be noted that, the increase of the collector width leads to an increase on the collector size as well as the increase of the collector cost. The large collector width remains a design issue depending on the type of application and the cost. The effect of the concentration ratio on the efficiency of the collector can be understood from Figure 7 and Figure 8. From Figure 7 it is clear that the increase of concentration ratio led to an increase on the heat gain and the temperature difference across the collector. The Figure shows that when the focus length value is fixed, the relationship between the collector efficiency and the concentration ratio is related reversely until specific point and then influenced directly. These relationships in Figure 8 contains a minimum value as (19.3%, 21.07 %, 22.35% and 23.33%) at concentration ratio of (69, 80, 103 and 148) in order with the focus length values (0.6m, 0.7m, 0.8m and 0.9m) respectively. Also, from this Figure it can be seen that the efficiency of the collector decreases with increasing the focus length.

The presented model is applied on available data for the city of Basra in the south of Iraq. Collector length and width and focus length are specified as 3.4m, (from 0.05m to 1m by step 0.05m increment), and 0.06m, respectively. Figure 9 shows the change in the efficiency of the collector throughout the day. During the months of the year. It is

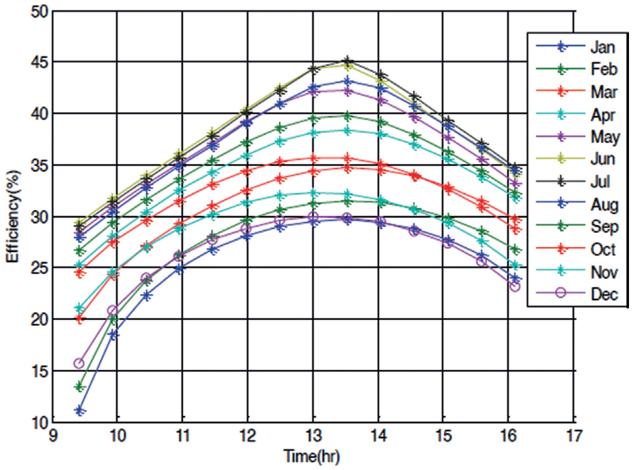

Figure 9. Collector efficiency.

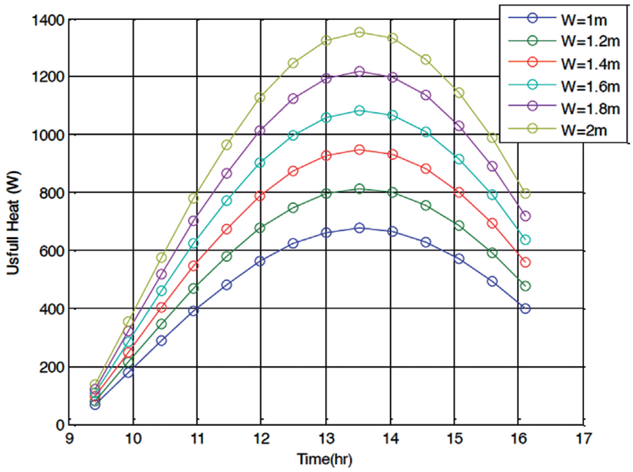

Figure 11. Effect the collector width on useful heat.

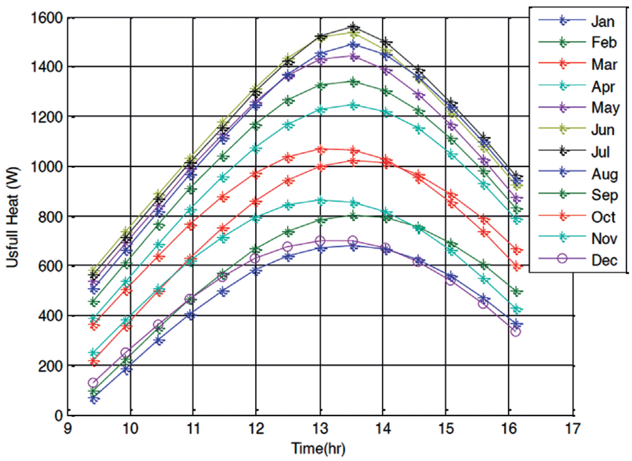

Figure 10. The useful heat.

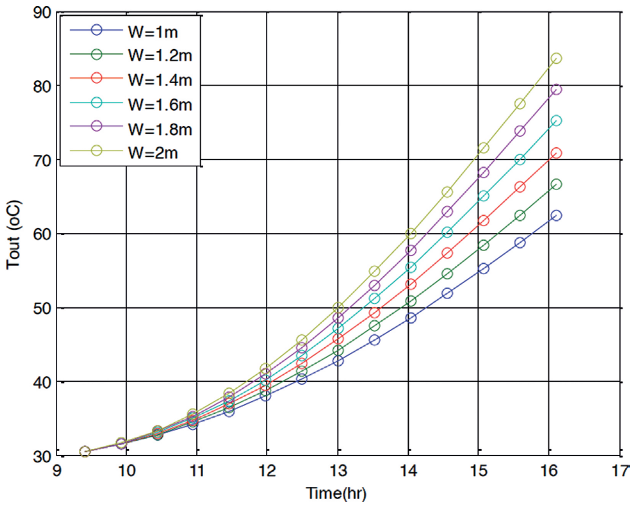

Figure 12. Effect the collector width on out temperature.

noticed that the highest efficiency for the collector is in the afternoon period for all months of the year. The June and July month witnessed the highest efficiency value of the collector, while the January and December recorded the lowest efficiency values.

Figure 10 shows that the useful energy gained from the collector changes during the day and its highest value is within the period from 13:00 to 14:00. On the other hand, the highest amount of useful energy gained was in the June and July months and the lowest value is the heat gained in the January and December. The gained heat plays an important role in the design of the collector, notice that the change in the efficiency of the collector shown in Figure 9 is a direct result of the effect and the disposal of the useful energy shown in this Figure.

From Figure 11, it can be observed that the width of the collector greatly influences the amount of useful heat. Whereas, when the width of the collector is doubled, the useful heat doubled too. This effect will reflect positively on the performance of the collector, as it leads to an increase in the output temperature of the collector when the diameter of the tube remains constant, this can be seen in Figure 6.

The effect of the length of the collector on both the heat gain and the output temperature can be seen in Figures 11 and 12, respectively. Likewise, from observation, doubling the length of the collector causes the useful heat and output temperature to double. The effect of the length of the collector on the heat gain of the collector can be seen in the Figure 13. Where the collector length correlated positively with the gained heat. Also, the effect of the collector length on the output temperature of the collector can be seen in Figure 14. The out temperature of the collector increased when the collector length increased too.

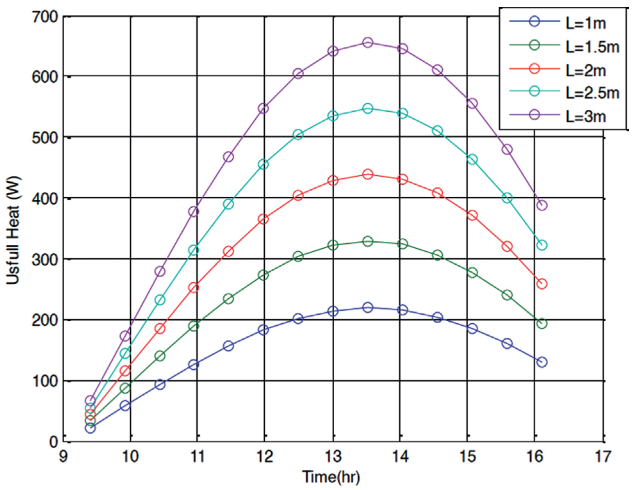

Figure 13. Effect the collector length on the useful heat.

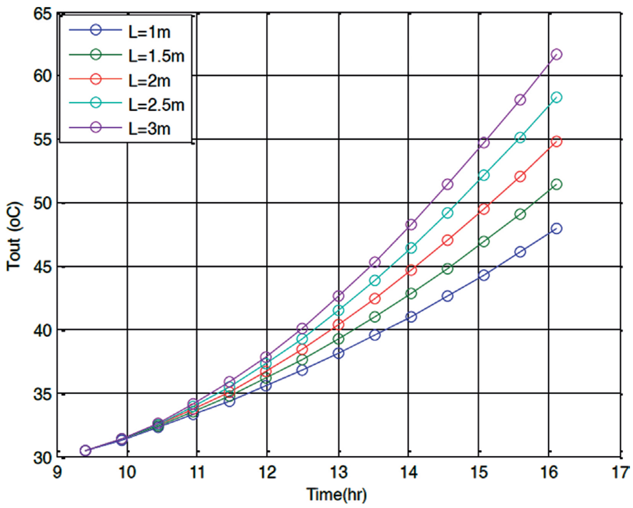

Figure 14. Effect the collector length on the out temperature.

CONCLUSION

In this study, a new mathematical model was derived to calculate the concentration ratio to model the heat absorbed from a parabolic solar collector. This method relied on simple mathematical and engineering foundations. The concentration ratio depends on two main variables in its calculation: the collector width (W) and focal length (P). The work included the application of the energy balance on working fluid passing through a pipe located along the focal of the collector. Based on the results presented above, it is possible to conclude the following:

- The increase in the width of the collector (W) leads to an increase in the concentration ratio (RC), while on the contrary is true when the focal length (P) increased.
- The collector efficiency minimum values were 19.3%, 21.07 %, 22.35% and 23.33% at concentration ratios of 69, 80, 103, and 148, in line with the focus length values of 0.6m, 0.7m, 0.8m and 0.9m, respectively.
- According to the proposed model, the maximum reached efficiency was 45% in July for Basra city.

NOMENCLATURE

Symbol	Definition
A1	Lower surface area of the pip, m^2
A2	Upper surface area of the pipe, m^2
Cp	Specific heat, KJ/kg.K
h	Heat transfer coefficient, W/m.K
I_c	Intensity solar radiation, W/m^2
I_N	Incident solar radiation, W/m^2
I_R	Reflect solar radiation, W/m^2
L	Pipe length, m
Nu	Nusselt Number
P	Focus length, m
Pr	Prantel number
q1	Heat flux from reflect solar radiation, W/m^2
q2	Heat flux from solar radiation, W/m^2
R	Pipe radius, m
Rc	Concentration ratio
T1	Inlet temperature for any element, °C
T2	outlet temperature for any element, °C
T_∞	Ambient temperature, °C
Tin	Pipe Inlet temperature, °C
Tout	Pipe outlet temperature, °C
W	Collector width, m
η	Collector efficiency
α	Collector rim angle, degree
β	Absorption coefficient
θ	Angle between the horizontal and the tangential line, degree
σ	Stefan Boltzmann constant being 5.67×10^{-8}, $W/m^2 K^4$

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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