

ON THE TURBULENT PRANDTL NUMBER IN STABLY STRATIFIED TURBULENCE BY SECOND ORDER MODELS

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ABSTRACT

The aim of this work is to investigate the behavior of the Turbulent Prandtl number by second order modeling of a stably stratified homogeneous sheared turbulence. By analytic solutions, we have confirmed the asymptotic equilibrium behavior of the turbulent Prandtl number. Then two between the most second order models of turbulence; the Classic Launder-Reece-Model and the sophisticated Craft Launder model are retained. A non dimensional form of transport equations have been obtained when non dimensional parameters are introduced to substitute second order moments. A numerical integration using the fourth order Runge kutta method has been conducted for different values of the gradient Richardson number R_i . In comparison with direct numerical simulation result's of Shih et al. the obtained results by the Craft Launder model has shown for the turbulent Prandtl number the best agreement at moderate values of gradient Richardson number $0.15 < R_i < 0.28$. The classic model has shown a great default for the different values of R_i . No any concordance with retained results of DNS has been obtained by this model. We show also that prediction of this model can be improved by introducing variation and optimization of model constants.

Keywords: *Prandtl Number, Stably Stratified Turbulence, Second Order Modeling*

INTRODUCTION

The problem of turbulent mixing in a stably stratified turbulence is a problem of often importance since stably stratified are present in the atmospheric boundary layer, oceans, lakes and many engineering flows. Homogenous turbulence in stratified shear flow has been investigated essentially through direct numerical simulations by several authors during the last three decades. Gerz et al. [1] and Holt et al. [2] have analyzed the behavior of a stratified homogeneous sheared turbulence as a function of the important parameter R_i . The gradient Richardson number R_i indicates the importance of stratification effect to shear effect. Shih et al. [3-4] have accorded much attention to the turbulent Prandtl number than the other works and their results are retained in this work. This number is a widely used parameter in stably stratified homogeneous turbulence.

More recently, Subhas and Derek [5] have argued these results and derived a new relationship for the turbulent Prandtl number in terms of the gradient Richardson number R_i , and between the turbulent kinetic energy and scalar variance. The second order modeling of stratified homogeneous turbulence remains to our sense an important approach of studying homogeneous turbulence since its application to the great number of engineering flows and industrial applications [6-7]. In a previous work, we are interested to the prediction of equilibrium states in stratified homogenous turbulence with vertical or inclined shear [8-9]. We were focused essentially to the asymptotic behavior of dimensionless kinematic parameters such as the component of the anisotropic tensor and the non dimensional shear number KS/ε .

However to our best knowledge not previous results have been dedicated to the prediction of the turbulent Prandtl number behavior using second order models. This simple and important goal constitutes the motivation of this paper which is organized as follows. In section 2 we present the mathematical formulation of a stratified homogeneous turbulence by the transport equation of second order moments of turbulence. Some analytical comments are than proposed for the turbulent Prandtl number Pr_t and the ratio L_M/L_E . In section 3 the second order modeling of transport equations is described and the retained second order models of turbulence are introduced. A non dimensional form of the transport equation is also obtained, parameterized by

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the gradient Richardson number R_i . In section 4 the numerical integrations conducted and the principal results are discussed. The section 5 the principal concluding of this work is dressed.

MATHEMATICAL FORMULATION GENERAL EQUATIONS

In the case of a stably stratified homogenous sheared turbulence, the basic transport equation for the components $\overline{u_i u_j}$ of the Reynolds stress, the turbulent kinetic energy, the turbulent scalar flux and scalar variance may be obtained by standard method [2]:

$$\frac{d}{dt} \overline{u_i u_j} = P_{ij} - B_{ij} + \Phi_{ij} - \varepsilon_{ij} \quad (1)$$

$$\frac{dK}{dt} = P - B - \varepsilon \quad (2)$$

$$\frac{d}{dt} (\overline{u_i \theta}) = P_{i\theta} - B_{i\theta} + \Phi_{i\theta} - \varepsilon_{i\theta} \quad (3)$$

$$\frac{d}{dt} \overline{\theta^2} = P_\theta - 2\varepsilon_\theta \quad (4)$$

In these equations terms noted P are production terms due to mean velocity and mean scalar gradients:

$$P_{ij} = -\overline{u_i u_k} \overline{U_{k,j}} - \overline{u_j u_k} \overline{U_{k,i}} \quad (5)$$

$$P = -\overline{u_i u_k} \overline{U_{k,i}} \quad (6)$$

$$P_{i\theta} = -\overline{u_i u_k} \overline{S_{T,k}} - \overline{u_j \theta} \overline{U_{i,k}} \quad (7)$$

$$P_\theta = -2\overline{u_2 \theta} \overline{S_T} \quad (8)$$

Φ_{ij} and $\Phi_{i\theta}$ are the pressure strain correlation and pressure scalar gradient correlation terms:

$$\Phi_{ij} = \overline{(p u_{i,j} + p u_{j,i})} \quad (9)$$

$$\Phi_{i\theta} = \frac{1}{\rho_0} \overline{p \theta_{,i}} \quad (10)$$

B_{ij} and B are buoyancy terms:

$$B_{ij} = \frac{g}{\rho_0} (\overline{u_i \theta} \delta_{j2} + \overline{u_j \theta} \delta_{i2}) \quad (11)$$

$$B = \frac{g}{\rho_0} \overline{u_i \theta} \delta_{i2} \quad (12)$$

$$B_{i\theta} = \frac{g}{\rho_0} \overline{\theta^2} \delta_{i2} \quad (13)$$

and ε are dissipation due to molecular effects terms:

$$\varepsilon_{ij} = 2\nu \overline{\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k}} \quad (14)$$

$$\varepsilon = \varepsilon_{ii}/2 = 2\nu \overline{\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_i}{\partial x_k}} \quad (15)$$

$$\varepsilon_{i\theta} = (\alpha + \gamma) \overline{u_{i,k} \theta_{,k}} \quad (16)$$

$$\varepsilon_\theta = 2\alpha \overline{\frac{\partial \theta}{\partial x_k} \frac{\partial \theta}{\partial x_k}} \quad (17)$$

Now, analytical comments are developed for transport equations (1) - (4). The behavior of the turbulent Prandtl number Pr_t and the ratio L_M/L_E will be investigated. In their direct numerical simulations, Holt et al. [2] have confirmed that the non linear and viscosity effects are very small comparing to productions terms at high shear ($\tau \gg$). If we take into account of this hypothesis in the transport equations (1) - (4), a linear set of differential equations are obtained and written in the following form:

$$\frac{d}{dt} \overline{u_1^2} = -2S \overline{u_1 u_2} \quad (18)$$

$$\frac{d}{dt} \overline{u_2^2} = 2\beta g \overline{u_2 \theta} \quad (19)$$

$$\frac{d}{dt} \overline{u_3^2} = 0 \quad (20)$$

$$\frac{d}{dt} \overline{u_1 u_2} = -S \overline{u_2^2} + \beta g \overline{u_1 \theta} \quad (21)$$

$$\frac{d}{dt} \overline{u_1 \theta} = -S_T \overline{u_1 u_2} - S \overline{u_2 \theta} \quad (22)$$

$$\frac{d}{dt} \overline{u_2 \theta} = -S_T \overline{u_2^2} + \beta g \overline{\theta^2} \quad (23)$$

$$\frac{d}{dt} \overline{\theta^2} = -2S_T \overline{u_2 \theta} \quad (24)$$

The Laplace transform is retained for integrating the linear differential system, the principal solutions are written as follows:

$$\begin{aligned} \overline{u_1 u_2} &= A_{12} Ch(\sqrt{R_i} \tau) + B_{12} Sh(\sqrt{R_i} \tau) + D_{12} Ch(2\sqrt{R_i} \tau) \\ &+ E_{12} Sh(2\sqrt{R_i} \tau) \end{aligned} \quad (25)$$

$$\overline{u_2 \theta} = C_{2\theta} Ch(2\sqrt{R_i} \tau) - B_{2\theta} Sh(2\sqrt{R_i} \tau) \quad (26)$$

$$2K = \overline{q^2} = \overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2} \quad (27)$$

$$\begin{aligned} \overline{q^2} = 2k &= A_q Ch(\sqrt{R_i} \tau) + B_q Sh(\sqrt{R_i} \tau) + D_q Ch(2\sqrt{R_i} \tau) \\ &+ E_q Sh(2\sqrt{R_i} \tau) + F_q \end{aligned} \quad (28)$$

$$\overline{\theta^2} = A_{\theta\theta} Ch(2\sqrt{R_i} \tau) + B_{\theta\theta} Sh(2\sqrt{R_i} \tau) \quad (29)$$

Where coefficients A, B, D are functions of initial values of turbulent parameters and the gradient Richardson number R_i . These solutions will now be used to study the asymptotic behavior (when $\tau \rightarrow \infty$) of the dimensionless parameters: the turbulent Prandtl number Pr_t and the ratio L_M/L_E . Firstly we deduce the expressions of these parameters:

$$\begin{aligned} Pr_t &= \frac{S_T}{S} \frac{\overline{u_1 u_2}}{u_2 \theta} \\ &= \frac{S_T}{S} \frac{A_{12} Ch(\sqrt{R_i} \tau) + B_{12} Sh(\sqrt{R_i} \tau) + D_{12} Ch(2\sqrt{R_i} \tau) + E_{12} Sh(2\sqrt{R_i} \tau)}{C_{2\theta} Ch(2\sqrt{R_i} \tau) - B_{2\theta} Sh(2\sqrt{R_i} \tau)} \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{L_M}{L_E} &= \frac{S_T}{S} \left(\frac{\overline{q^2}}{\overline{\theta^2}} \right)^{1/2} \\ &= \frac{S_T}{S} \left(\frac{A_q Ch(\sqrt{R_i} \tau) + B_q Sh(\sqrt{R_i} \tau) + D_q Ch(2\sqrt{R_i} \tau) + E_q Sh(2\sqrt{R_i} \tau) + F_q}{A_{\theta\theta} Ch(2\sqrt{R_i} \tau) + B_{\theta\theta} Sh(2\sqrt{R_i} \tau)} \right)^{1/2} \end{aligned} \quad (31)$$

And at high shear ($\tau \gg$), we obtain respectively:

$$(Pr_t)_\infty = \lim_{\tau \rightarrow \infty} \left(\frac{S_T}{S} \frac{\overline{u_1 u_2}}{u_2 \theta} \right) = \frac{S_T}{S} \frac{D_{12} + E_{12}}{C_{2\theta} - B_{2\theta}} \quad (32)$$

$$\left(\frac{L_M}{L_E} \right)_\infty = \lim_{\tau \rightarrow \infty} \left(\frac{S_T}{S} \left(\frac{\overline{q^2}}{\overline{\theta^2}} \right)^{1/2} \right) = \frac{S_T}{S} \frac{(D_q + E_q)^{1/2}}{(A_{\theta\theta} + B_{\theta\theta})^{1/2}} \quad (33)$$

We conclude that at high shear ($\tau \gg \epsilon$), the two dimensionless scalar parameters tend to equilibrium states functions of the initial values of turbulent parameters and the gradient Richardson number R_i . This analytic solution is important since it confirms that at equilibrium, buoyancy terms equilibrate production terms. It confirms also the result of direct numerical simulation of Shih et al. [4] and the study of Subhas and Derek [5]. This approach is only a qualitative one and a more quantitative study of the evolution of the turbulent Prandtl number Pr_t and the ratio L_M/L_E will be addressed in the following section when second order modeling of transport equations is retained.

SECOND ORDER MODELING

At this step of our work and with the aim of obtaining a closed set of equations, the second order modeling is the approach retained here. Second order models are retained for nonlinear terms of pressure strain correlation, pressure scalar gradient correlation and transport equation of dissipation.

In a stratified shear flow, the correlations Φ_{ij} and $\Phi_{i\theta}$ are classically [11] decomposed on three contributions:

$$\Phi_{ij} = \Phi_{ij}^1 + \Phi_{ij}^2 + \Phi_{ij}^3 \quad (34)$$

$$\Phi_{i\theta} = \Phi_{i\theta}^1 + \Phi_{i\theta}^2 + \Phi_{i\theta}^3 \quad (35)$$

Where the contributions 1 are the return-to-the isotropy terms, the terms 2 represent the interaction between mean and turbulent flows. The terms 3 are terms due to buoyancy.

Two between the most known second order models are retained for the pressure strain correlation and the pressure scalar gradient correlation and the evolution equation of the dissipation rate ϵ . The classic second order model of Launder, Reece and Rodi (LRR) [12] in one hand and the sophisticated model of Craft and Launder (CL) [13] have been retained. For its success in many different applications of stratified homogeneous shear flow [8-10], the Zeman Lumley model is retained for the third contributions of the pressure strain correlation and pressure scalar gradient correlation:

$$\Phi_{ij}^3 = C_3 \left(\frac{1}{\rho_0} (g_i \overline{u_j \theta} + g_j \overline{u_i \theta} - \frac{2}{3} \delta_{ij} \frac{g_m}{\rho_0} \overline{u_m \theta}) \right) \quad (36)$$

$$\Phi_{i\theta}^3 = C_{\theta 3} \frac{1}{\rho_0} g_i \overline{\theta^2} \quad (37)$$

Where $C_3 = 0.5$ and $C_{\theta 3} = 0.5$.

Introducing the non dimensional time $\tau = St$, a modeled form retained for the transport equations of the mentioned parameters b_{ij} , SK/ϵ , Pr_t and L_M/L_E are obtained by:

$$\begin{aligned} \frac{d}{d\tau} b_{ij} = & - \left(b_{i2} + \frac{\delta_{i2}}{3} \right) \delta_{j1} - \left(b_{j2} + \frac{\delta_{j2}}{3} \right) \delta_{i1} + \frac{\beta g}{2SK} (\overline{u_i \theta} \delta_{j2} + \overline{u_j \theta} \delta_{i2}) + \frac{1}{2SK} \Phi_{ij} - \frac{1}{3} \frac{\epsilon}{SK} \delta_{ij} \\ & + \left(b_{ij} + \frac{\delta_{ij}}{3} \right) \left(2b_{12} - \frac{\beta g}{SK} \overline{u_2 \theta} + \frac{\epsilon}{SK} \right) \end{aligned} \quad (38)$$

$$\frac{d}{d\tau} \left(\frac{SK}{\epsilon} \right) = (1 + C_{\epsilon 1} - C_{\epsilon 3}) \left(\frac{SK}{\epsilon} \right) \frac{2R_i b_{12}}{Pr_t} + (C_{\epsilon 1} - 1) 2b_{12} \left(\frac{SK}{\epsilon} \right) - 1 + C_{\epsilon 2} \quad (39)$$

$$\frac{d}{d\tau} Pr_t = \left[-\left(b_{22} + \frac{1}{3}\right) + \frac{\Phi_{12}}{2KS} \right] \frac{Pr_t}{b_{12}} + R_i \frac{\overline{\theta u_1}}{\theta u_2} + \left[b_{22} + \frac{1}{3} - \frac{R_i}{(L_M/L_E)^2} - \frac{\Phi_{2\theta}}{2KS_T} \right] \frac{(Pr_t)^2}{b_{12}} \quad (40)$$

$$\frac{d}{d\tau} \left(\frac{L_M}{L_E} \right) = (R_i Pr_t - 1) b_{12} \left(\frac{L_M}{L_E} \right) + \left(\frac{1}{r} - \left(\frac{K}{2} \right)^{1/2} \right) \left(\frac{\varepsilon}{SK} \right) \left(\frac{L_M}{L_E} \right) + \frac{b_{12}}{Pr_t} \left(\frac{L_M}{L_E} \right)^3 \quad (41)$$

$$\frac{d}{d\tau} \left(\frac{\overline{\theta u_1}}{\theta u_2} \right) = \left(\frac{\Phi_{1\theta}}{2KS_T b_{12}} - 1 \right) Pr_t + \left(\frac{\overline{\theta u_1}}{\theta u_2} \right) \left[b_{22} + \frac{1}{3} - \frac{R_i}{(L_M/L_E)^2} - \frac{\Phi_{2\theta}}{2KS_T} \right] \frac{Pr_t}{b_{12}} \quad (42)$$

At this step, a numerical integration of the differential equations is started. Obtained results will be discussed in the following sections.

NUMERICAL INTEGRATION AND RESULTS

Two non linear systems of seven differential equations are obtained corresponding to the two retained models. The numerical integration is advanced in time using the fourth order Runge-Kutta method. Numerical integration has been advanced to long time evolution $\tau = 12$. The principal results are now discussed : A plot of the turbulent Prandtl number Pr_t for weak stratification, $R_i = 0.05, 0.06$ and 0.1 , is shown in Figures 1. The Craft Launder model shows an overestimation of about 50% direct numerical simulations values. The LRR model indicates an under estimation of these values.

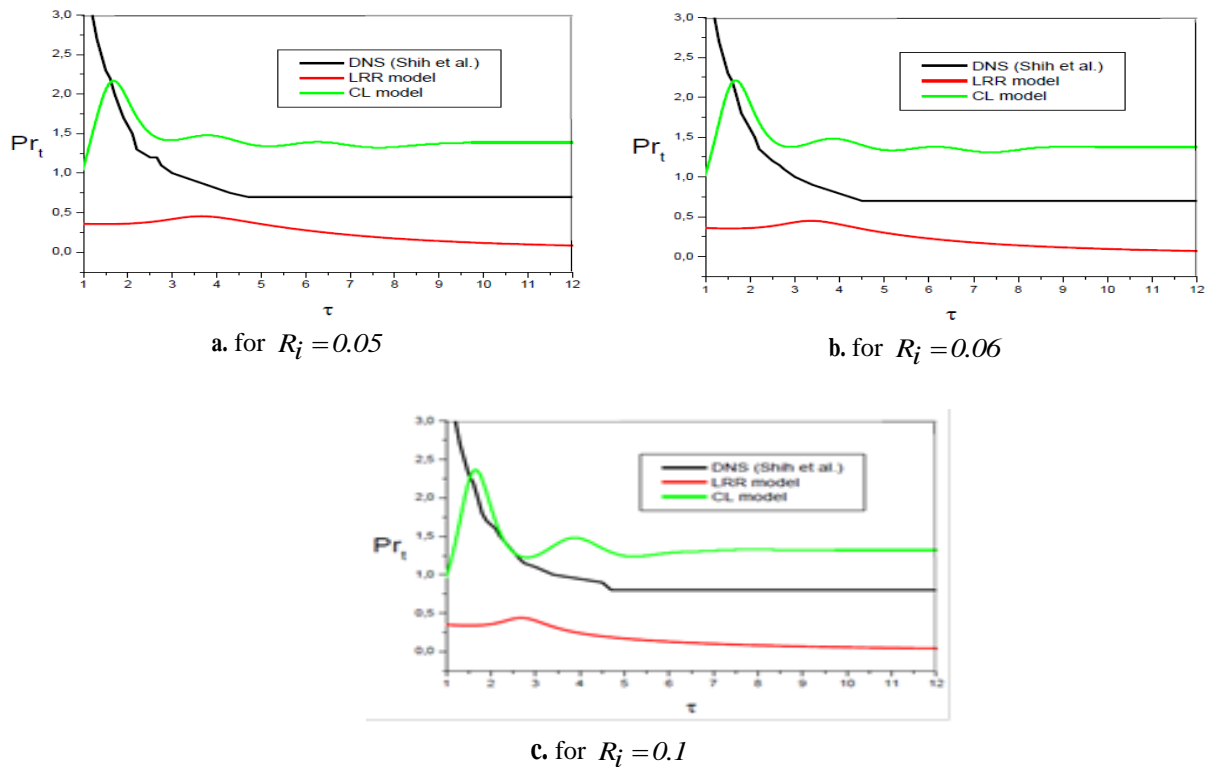


Figure 1. Time evolution of Pr_t for Data of Shih et al. and LRR and CL models

The estimation of the CL model becomes better for moderate values of R_i equals respectively to 0.15, 0.18 and 0.25. The differences between predictions of CL model and direct numerical simulations values is less important than in the case of weak values of R_i as indicated in Figures 2. An over estimation of 10% the direct numerical values for $R_i = 0.18$ and an under estimation of 10% values of direct numerical simulation for $R_i = 0.25$ has been observed by CL model. The LRR model does not indicate interesting behavior for the moderated or

weak stratification for the turbulent Prandtl number. A great difference between LRR model predictions and the values of the direct numerical simulation is observed for both weak and moderate stratification.

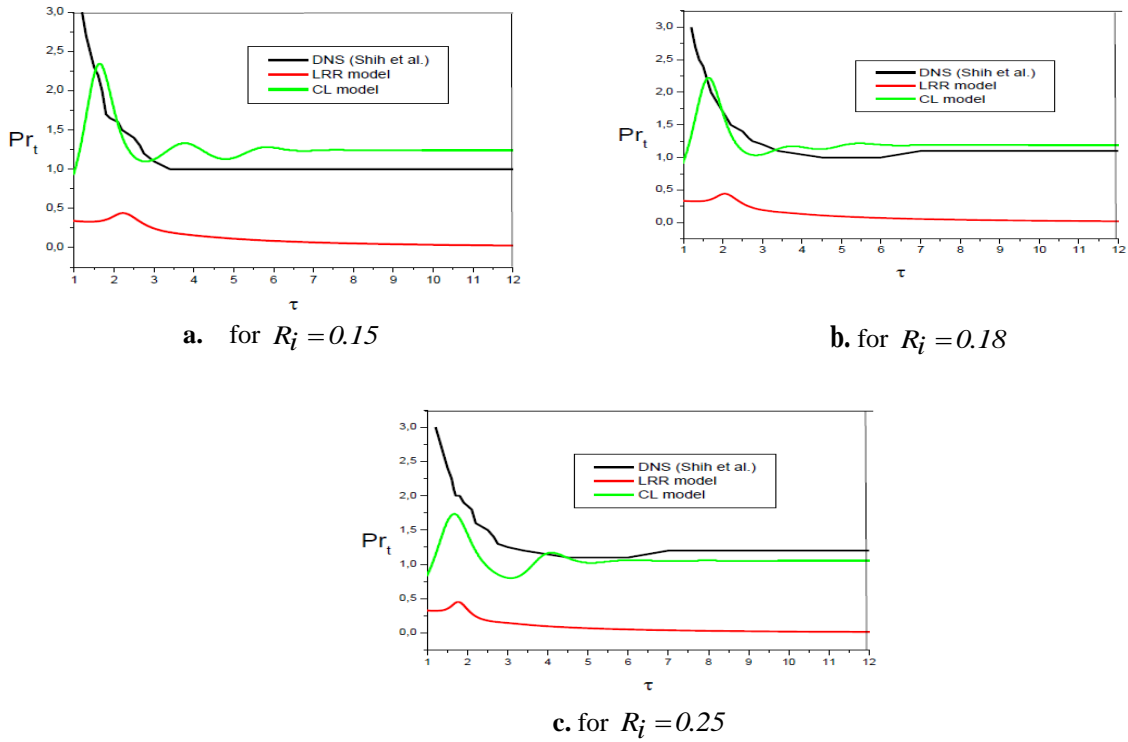


Figure 2. Time evolution of Pr_t for Data of Shih et al. and LRR and CL models

Figures 3 indicate plot of Prandtl number at high values of the gradient Richardson number $R_i = 0.37, 0.4$ and 0.6 . Here no concordance between models predictions and values of direct numerical simulations has been observed. An under estimation of 50% values of DNS of Shih et al. has been observed for $R_i = 0.37$ and $R_i = 0.4$ by Craft Launder model whereas no any concordance has been observed between the LRR model and values of direct numerical simulations of Shih et al. [4].

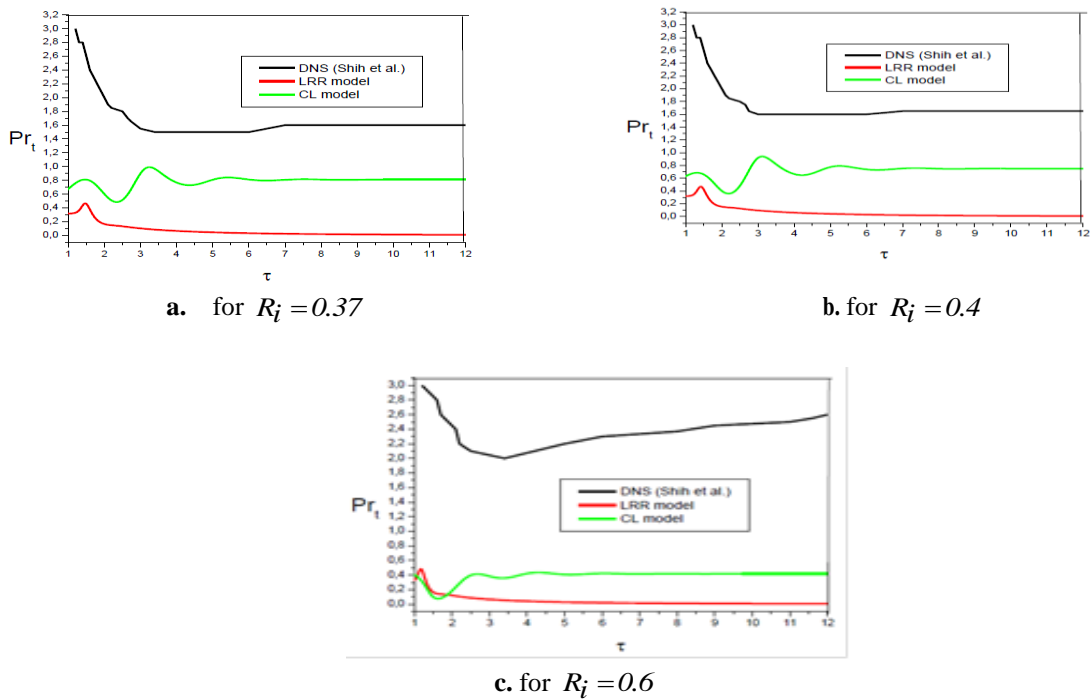


Figure 3. Time evolution of Pr_t for Data of Shih et al. and LRR and CL models

Table 1 indicates equilibrium values of turbulent Prandtl number by models and direct numerical simulations. Here also acceptable concordance with values of DNS is obtained for moderate values of the gradient Richardson number R_i by the Craft Launder model. An over estimation and an underestimation is obtained by this model for weak and high stratification respectively.

Table 1. Equilibrium values of Pr_t for different values of R_i

R_i	$(Pr_t)_\infty$ (DNS (Shih et al.))	$(Pr_t)_\infty$ (LRR Model)	$(Pr_t)_\infty$ (CL Model)
0.05	0.7	0	1.39
0.06	0.7	0	1.38
0.1	0.8	0	1.32
0.15	1	0	1.24
0.18	1.05	0	1.19
0.25	1.25	0	1.05
0.37	1.6	0	0.81
0.4	1.7	0	0.75
0.6	-	0	0.418

To improve performance of the LRR model, correction to model coefficient is retained by many authors. The first coefficient C_I of the return to isotropy model of the pressure strain correlation Φ_{ij} is generally the coefficient concerned in this modification. Here, we are interested to modify this coefficient C_I with small variations around the known value $C_I = -1.8$. The equilibrium values of Pr_t obtained as function of C_I are presented on the following table:

Table 2. Equilibrium values of Pr_t for different values of C_I for $R_i = 0.05$ and $R_i = 0.37$

	$(Pr_t)_\infty$	
	$R_i = 0.05$	$R_i = 0.37$
LRR $C_I = -0.35$	0.504	0.137
LRR $C_I = -0.4$	0.423	0.109
LRR $C_I = -0.5$	0.37	0
LRR $C_I = -1$	0.177	0
LRR $C_I = -1.8$	0	0
LRR $C_I = -2$	0	0
DNS	0.7	1.6

Another optimization which we propose here is for the coefficient $C_{\theta I}$ of the return to isotropy of the model of pressure scalar gradient correlation. We obtain the following table of equilibrium values of Pr_t for different values of this coefficient:

We see that the equilibrium value of Pr_t is very sensitive to variation of the coefficient C_I for the weak value 0.05 of the gradient Richardson number R_i . When C_I equal -0.5 and -1, a better approximation of the DNS value is observed by LRR model. The best approximation is observed for $C_I = -0.5$. For $R_i = 0.37$ no improve of the LRR model has been observed. The same observation is also available for variation of the coefficient $C_{\theta I}$ of the return to isotropy model of pressure scalar gradient correlation. Variation of model coefficient can improve prediction of LRR model only for weak values of R_i .

Table 3. Equilibrium values of Pr_t for different values of $C_{\theta 1}$ for $R_i = 0.05$ and $R_i = 0.37$

	$(Pr_t)_\infty$	
	$R_i = 0.05$	$R_i = 0.37$
LRR $C_{\theta 1} = -8$	0.673	0
LRR $C_{\theta 1} = -6$	0.338	0
LRR $C_{\theta 1} = -3.6$	0.102	0
LRR $C_{\theta 1} = -3.2$	0	0
LRR $C_{\theta 1} = -2.9$	0	0
DNS	0.7	1.6

It is essentially here to precise that coefficients C_1 and $C_{\theta 1}$ of return to isotropy terms are generally the only coefficients which we can modify. The values of the others coefficients of the linear terms of pressure strain correlation and pressure scalar gradient correlations are exact. They cannot be modified since they are submitted in addition to kinematic constraints (continuity and symmetry...) to the strong form of realisability condition [13].

Figures 4 show the plot of the ratio L_M/L_E . No concordance has been observed for the case of weak stratification between predictions models and the direct numerical simulation values. An acceptable agreement between predictions models and DNS at high values of the gradient Richardson number $R_i = 0.37, 0.4$ and 0.6 . More precisely, for different values of gradient Richardson number R_i , the ratio L_M/L_E reaches a constant value from τ greater than 6. For the Craft Launder model, we observe in table 4 an over estimation of asymptotic values of about 50% for the values $R_i = 0.37$ and 0.4 . An excellent agreement with the value of DNS of Shih et al. has been observed for the value 0.6 of the gradient Richardson number.

The default of the LRR model is here clear also no concordance has been observed between LRR model and DNS values.

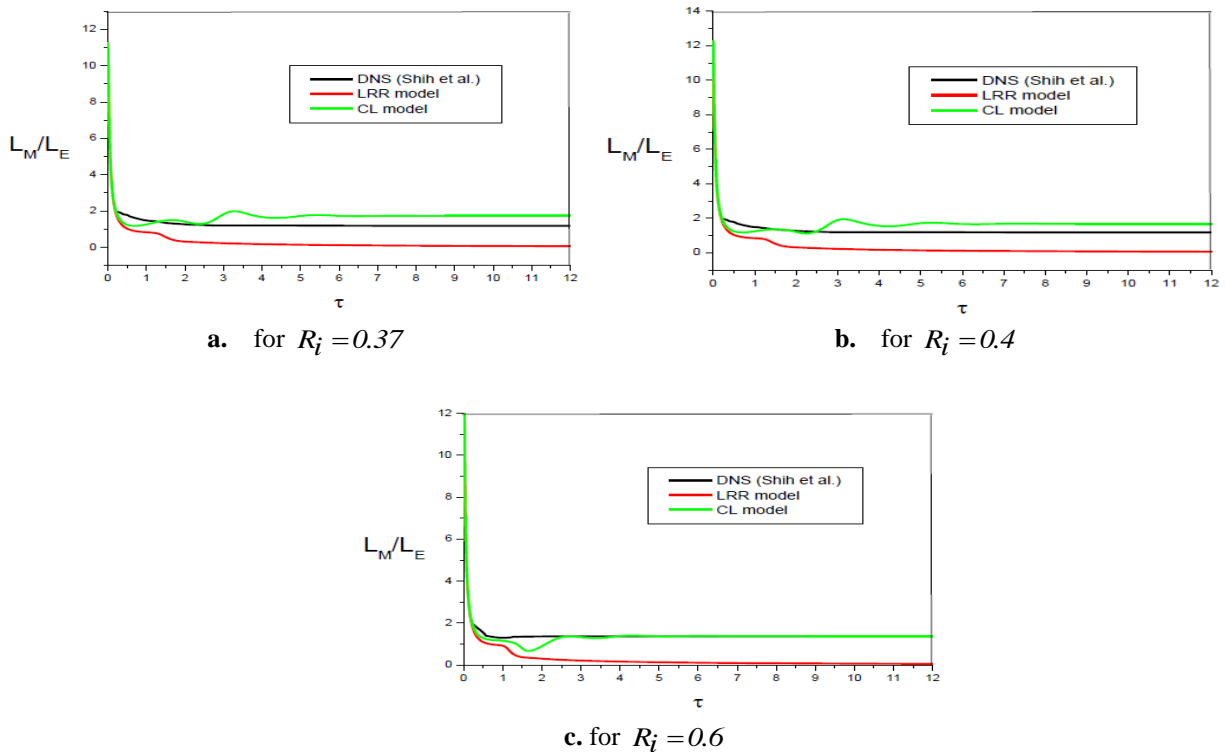


Figure 4. Time evolution of L_M/L_E for Data of Shih et al. and LRR and CL models

Table 4. Equilibrium values of L_M/L_E for different values of R_i

R_i	$(L_M/L_E)_\infty$ (DNS (Shih et al.))	$(L_M/L_E)_\infty$ (LRR model)	$(L_M/L_E)_\infty$ (CL model)
0.37	1.12	0	1.73
0.4	1.14	0	1.68
0.6	1.36	0	1.38

CONCLUDING REMARKS

In this work, we are interested to the study of the evolution of scalar parameters in stably stratified homogeneous sheared turbulence using second order models of turbulence. The turbulent Prandtl number is the principal parameter concerned. The ratio L_M/L_E of mechanical length scale to scalar length scale is also investigated. At a first step, we have confirmed by analytical linear solutions, deduced from Laplace transform and available only at high shear that the two mentioned dimensionless parameters have a general tendency to asymptotic equilibrium states, observed by the DNS of Shih et al. In the second step, two second order models have been retained for pressure strain correlation and pressure scalar gradient correlation to model transport equations. The fourth order Runge-Kutta method has been retained for integrating two non linear systems of seven differential equations.

The obtained results have shown an acceptable concordance between the Craft Launder model and DNS values at moderate values of the gradient Richardson number 0.15, 0.18 and 0.25. A great default of the LRR model has been observed here with the values of the DNS of Shih et al. It confirms the default of this model in stratified homogeneous turbulence observed in our previous work [8]. To improve its prediction, we make variations of the coefficients C_I and $C_{\theta I}$ of the terms of return to isotropy of pressure strain correlation and pressure scalar gradient correlation. Only at weak value of the gradient Richardson number $R_i = 0.05$ a better approximation of the DNS values of Shih et al. [4] has been observed. For the moderate value of gradient Richardson number $R_i = 0.37$ no improve of the LRR model has been observed for variations of coefficients C_I and $C_{\theta I}$. To improve predictions of LRR model and CL model, at high gradient Richardson number, we think that introduction of correction function by analogy as that introduced by Speziale et al. [14] and Younis et al. [15] in the case of turbulence, with rotation can make an important direction of investigation in a future work.

NOMENCLATURE

$b_{ij} = (\overline{u_i u_j} / 2K) - (\delta_{ij} / 3)$	Reynolds stress anisotropy tensor
g_i	Gravitational acceleration [m/s ²]
K	Turbulent kinetic energy [m ² /s ²]
$L_M = (2K)^{1/2} / S$	Mechanical length scale [m]
$L_E = (\overline{\theta^2})^{1/2} / S_T$	Scalar length scale [m]
P	Production due to mean kinematic and scalar gradients
P	Pressure [N/m ²]
p	Fluctuation of the pressure [N/m ²]
Pr_t	Turbulent Prandtl number
$R_i = g \beta S_T / S^2$	Dimensionless Richardson number
S	Shear rate [s ⁻¹]
S_{ij}	The mean strain rate tensor [s ⁻¹]
S_T	Mean scalar gradient [°C/m]
t	Time [s]
T	Temperature [°C]

u_i	i-th component of the fluctuating velocity [m/s]
\overline{U}_i	i-th component of mean velocity [m/s]
$\overline{u_i u_j}$	Reynolds stress tensor [m ² /s ²]
$\overline{u_i \theta}$	Turbulent scalar flux [m ³ /s]
W_{ij}	Mean vorticity tensor [s ⁻¹]
x_i	Component of an orthonormal Cartesian coordinate system [m]
Greek Symbols	
α	Thermal diffusivity of fluid [m ² /s]
β	Thermal expansion coefficient [K ⁻²]
δ_{ij}	Kronecker Symbol
ε	Dissipation due to molecular effects [m ² /s ³]
τ	Non dimensional time ($\tau = St$)
θ	Fluctuation of the scalar [°C]
$\overline{\theta^2}$	Temperature variance [°C ²]
μ	Dynamic viscosity [kg.m ⁻¹ .s ⁻¹]
ν	Kinematic viscosity [m ² /s]
ρ_0	Reference density of the fluid [kg/m ³]
Φ_{ij}	Pressure-strain correlation [m ² /s ³]
$\Phi_{i\theta}$	Pressure-temperature gradient correlation [°C.m/s ²]

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